Aerospace Science and Technology 71 (2017) 12–24

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Adaptive unscented Kalman filter based on maximum posterior and random weighting

Zhaohui Gao [a,*], Dejun Mu [a], Shesheng Gao [a], Yongmin Zhong [b], Chengfan Gu [b]

[a] School of Automatics, Northwestern Polytechnical University, Xi'an 710072, China
[b] School of Engineering, RMIT University, Bundoora, VIC 3083, Australia

ARTICLE INFO

Article history:
Received 8 November 2016
Received in revised form 15 June 2017
Accepted 17 August 2017
Available online 12 September 2017

Keywords:
Unscented Kalman filter
System noise statistics
Maximum posterior estimation
Random weighting

ABSTRACT

The unscented Kalman filter (UKF) is an effective technique of state estimation for nonlinear dynamic systems. However, its performance depends on prior knowledge on system noise. If the characteristics of system noise are unknown or inaccurate, the filtering solution may be biased or even divergent. This paper presents a new maximum posterior and random weighting based adaptive UKF (MRAUKF) by combining the concepts of maximum posterior and random weighting to overcome this limitation. The proposed MRAUKF computes noise statistics based on the maximum posterior principle, and subsequently adopts the random weighting concept to optimize the obtained maximum posterior estimations by online adjusting the weights on residuals. The maximum posterior and random weighting estimations of noise statistics are established to online estimate and adjust system noise statistics, leading to the improved filtering robustness. Simulation and experimental results demonstrate that the proposed MRAUKF outperforms the classical UKF and adaptive robust UKF in the presence of uncertain system noise statistics.

1. Introduction

Considerable research efforts have been dedicated to discrete-time filtering for nonlinear dynamic system [1–5]. As a relatively new method, the unscented Kalman filter (UKF) is an improvement to the extended Kalman filter (EKF) to generalize the traditional Kalman filter for both linear and nonlinear systems. Given the fact that the approximation of a nonlinear distribution is easier than that of a nonlinear function or transformation [2], UKF can approximate the posterior mean and covariance of system state vector for any Gaussian nonlinear system in third-order accuracy, whereas EKF in first-order accuracy [4]. This was validated by Allotta et al. [6,7], where the performances of UKF and EKF were compared for an autonomous underwater vehicle (AUV) navigation system. Further, as its estimation characteristic is not affected by the level of nonlinearity, UKF is particularly suitable for a strongly nonlinear system, which is commonly used in many engineering fields such as integrated navigation [5], AUV navigation [6,7], system identification [8] and target tracking [9].

However, the UKF performance depends on the accurate characterization of noise in system state and measurement. If the statistical characteristics of system noise are not known exactly, the filtering solution will be biased or even divergent [10–12]. In order to improve the UKF robustness, it is absolutely necessary to leverage the information obtained in the filtering process to online estimate and update noise statistics to resist the disturbance of uncertain system noise on system state estimation.

Research efforts have been dedicated to the improvement of the UKF adaptability and robustness against uncertain statistics of system noise. Cho and Choi reported a sigma point based receding-horizon Kalman filter (SPRHKF). This filter improves the UKF using the receding horizon strategy to adaptively resist model uncertainty and temporarily unknown sensor bias [13]. However, due to the use of a finite impulse response structure, the filtering convergence is poor. Cho and Kim developed an adaptive filter by combining the UKF and SPRHKF through an interactive multi-model estimator to overcome the shortcomings of both UKF and SPRHKF [14]. However, it causes an expensive computational load, unable to achieve the real-time performance. Song and Han reported a method to dynamically adjust the noise statistics by minimizing a cost function defined by the difference between the time-varying and average innovation covariances [10]. However, because of a great amount of derivative calculations involved, this method is also difficult to achieve the real-time performance. Wang studied an adaptive robust UKF (ARUKF) to weaken the perturbance on the Kalman filtering accuracy due to system model uncertainty [15].

* Corresponding author.
E-mail address: 88556522@qq.com (Z. Gao).

http://dx.doi.org/10.1016/j.ast.2017.08.020
1270-9638/

However, as the adaptive factors and equivalent weighting factors in the ARUKF are determined from empirical evidence, this method does not address the UKF limitation fundamentally.

Research efforts have also been reported focusing on online estimation rather than direct correction of noise statistics. These studies are mainly based on the assumption that the state and measurement disturbances involved in a dynamic system are additive and subject to a Gaussian distribution with zero mean. Thus, the problem of noise description is degraded to an estimation problem of covariance matrices of multi-variable Gaussian distributions. The innovation based adaptive estimation (IAE) and residual based adaptive estimation (RAE) are two typical methods for online estimation of noise covariances [16–19], where the adaptive estimation is directly applied to the covariance matrices of measurement noise and system noise according to variations in the residual or innovation sequence within a moving window. However, because of the complexity of nonlinear systems, the existing studies on the IAE and RAE are mainly dominated by linear systems, while the related research on nonlinear systems is still very limited [20]. The correlation method, which is commonly used for estimation in time series analysis, estimates the covariance matrices of system noise via the sample auto-correlations between innovation sequences [21,22]. However, this method is mainly suitable for a linear system with constant coefficients, unsuitable for any nonlinear filter. The covariance matching technique is an adaptive algorithm to compute the estimates of process noise covariance and measurement noise covariance at every sampling point by matching the elements of innovation or residual's covariance matrices to their theoretical values [12,23]. However, this method may yield the biased covariance estimate if the covariance matching involves error [24].

Maximum posterior (MP) is an estimation approach based on deep-rooted Bayesian formalism to estimate parameters through the maximization of their posteriori probability densities [25]. Assume that θ is the unknown random variable, z is the measurement related to θ, and $p(\theta|z)$ represents the conditional density function of θ with respect to z, which is called the posteriori probability density of θ. The MP estimation of random variable θ can be constructed by

$$\hat{\theta}_{MP} = \arg\max_{\theta} p\left(\theta|z\right) \tag{1}$$

The MP estimation has been proved to be the minimum variance estimator if system state vector and measurement vector obey the Gaussian distribution. As it allows the incorporation of prior knowledge on system model noise in state parameter estimation, the MP estimation can be used to construct an adaptive UKF [26]. However, as all residuals from historical epochs equivalently contribute to the evaluation of system noises, the noise statistics obtained by the MP estimation may not accurately describe the real characteristics of system noises, leading to biased estimation results. Zhao et al. [27] studied an adaptive UKF by combining the MP estimation with exponential weighting. This method develops a constant noise statistics estimator using the MP estimation and subsequently extends it to the time-varying noise case via exponential weighting. However, this constant noise statistics estimator still suffers from the aforementioned problem of the MP estimation. Further, the forgetting factor used in this method has to be determined by empiricism, limiting the improved filtering performance.

Random weighting is an emerging computational method in statistics [28,29]. Assume that $X_1, X_2, \cdots, X_n$ are independent and identically distributed random variables with common distribution function $F(x)$, and the corresponding empirical distribution function is

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{(X_i \le x)} \tag{2}$$

The random weighting estimation of $F_n(x)$ can be defined as

$$H_n(x) = \sum_{i=1}^{n} v_i I_{(X_i \le x)} \tag{3}$$

where $I_{(X_i \le x)}$ is the indicator function, i.e., $I_{(X_i \le x)} = \{^{1\ X_i \le x}_{0\ X_i > x}$, and random vector $(v_1, v_2, \cdots, v_n)$ is subject to Dirichlet distribution $D\ (1, 1, \ldots, 1)$, that is, $\sum_{i=1}^{n} v_i = 1$ and the joint density function of $(v_1, v_2, \cdots, v_{n-1})$ is

$$f(v_1, v_2, \cdots, v_n) = \Gamma(n) \quad \text{and}$$
$$(v_1, v_2, \cdots, v_{n-1}) \in s_{n-1}$$
$$= \left\{ (v_1, v_2, \cdots, v_{n-1}) : v_i \ge 0, \sum_{i=1}^{n-1} v_i \le 1 \right\}$$

The random weighting method can provide an unbiased estimation, and is simple in computation and suitable for large samples. Given these merits, the random weighting method has been widely used in solving various problems, such as parameter estimation, M-test in linear models, multi-sensor data fusion, and dynamic navigation and positioning [30–34].

This paper presents a new MP and random weighting based adaptive UKF (MRAUKF) to address the problem of the classical UKF in requiring accurate statistical characteristics of system noise. The proposed MRAUKF combines the MP principle with random weighting concept to dynamically estimate and adjust system noise statistics by leveraging the information obtained in the filtering process for improving the filtering performance. It establishes MP estimations for the covariances of process noise and measurement noise. Subsequently, random weighting estimations are established to optimize the MP estimations by online adjusting the weights on residuals. Simulations, experiments and comparison analysis have been conducted to comprehensively evaluate the performance of the proposed MRAUKF.

Different from Zhao's method, the proposed MRAUKF adopts the random weighting concept to fundamentally solve the problem of the MP estimation by dynamically adjusting the weights on residuals to improve the estimation accuracy of system noise statistics. Further, it also overcomes the limitation of Zhao's method in determination of the forgetting factor by empiricism, leading to the improved filtering performance.

2. Analysis of classical UKF

Consider the following discrete nonlinear system with linear measurement equation

$$\boldsymbol{X}_k = f(\boldsymbol{X}_{k-1}) + \boldsymbol{w}_k \tag{4}$$

$$\boldsymbol{Z}_k = \boldsymbol{H}_k \boldsymbol{X}_k + \boldsymbol{v}_k \tag{5}$$

where $\boldsymbol{X}_k$ is the state vector at time k, $\boldsymbol{Z}_k$ is the measurement vector, $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are the additive process noise and measurement noise, $f(\cdot)$ is a nonlinear function describing the process model, and $\boldsymbol{H}_k$ is the measurement matrix.

Process noise $\boldsymbol{w}_k$ and measurement noise $\boldsymbol{v}_k$ are assumed to be uncorrelated zero-mean Gaussian white noises with the following covariances

$$E\left[\boldsymbol{w}_k \boldsymbol{w}_j^{\mathrm{T}}\right] = \boldsymbol{Q}\,\delta_{kj}, \qquad E\left[\boldsymbol{v}_k \boldsymbol{v}_j^{\mathrm{T}}\right] = \boldsymbol{R}\delta_{kj} \quad \text{and} \quad E\left[\boldsymbol{w}_k \boldsymbol{v}_j^{\mathrm{T}}\right] = 0 \tag{6}$$