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Structural reliability sensitivity analysis based on classification of model output

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ABSTRACT

In structural reliability analysis, sensitivity analysis can be used to measure how the input variable influences the failure of structure. In this work, a new reliability sensitivity analysis method is proposed. In the proposed method, the model output is separated into two classes (failure domain and safe domain). The basic idea is that if the failure-conditional probability density function of input variable is significantly different from its unconditional probability density function, then the input variable is sensitive to the failure of structure. The proposed reliability sensitivity indices contain both individual sensitivity index and interaction sensitivity index. The individual sensitivity index can measure the individual effect of input variable on the failure of structure. The asymmetrical interaction sensitivity index can measure how one input variable influences the effect of another input variable on the failure of structure. Additionally, the meanings of the proposed reliability sensitivity indices are also interpreted explicitly, and a data-driven estimation method is also proposed to estimate the proposed reliability sensitivity indices. Finally, a numerical example and two engineering examples are presented to illustrate the rationality of the proposed sensitivity indices and the feasibility of the proposed estimation method.

1. Introduction

In practical engineering, such as aeronautical engineering and civil engineering, computer models are widely used to predict the real structural response. Uncertainties are often encountered in these computer models [1–3], which will lead to uncertainty performance. Uncertainty analysis has been widely used to help researchers evaluate the degree of confidence of model results and assess the risk [4]. Unfortunately, most applications of uncertainty analysis just provide the information of the uncertainty of model output, but do not give information on how the uncertainty of model output can be apportioned to the uncertainty of model inputs [5,6]. Thus, researchers cannot decide how to allocate resources to input factors so as to reduce the uncertainty of model output most effectively [7]. Sensitivity analysis can help researchers understand "*how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input*" [8]. Sensitivity analysis can be generally classified into local sensitivity analysis (LSA) and global sensitivity analysis (GSA). Local sensitivity measure is also known as One At a Time (OAT) measure, which is usually based on the estimation of partial derivative. It can measure the effect of one input variable on the model output while other input variables are fixed at nominal values [9]. The shortage of LSA is that it only provides sensitivity information at the nominal point where the derivative is calculated and cannot detect the interaction among different input variables. On the contrary, GSA can measure the effect of input variables on the model output in their entire distribution ranges and provide the interaction effect among different input variables [10]. In risk assessment, GSA is also known as uncertainty importance analysis, which is utilized to identify the most critical and essential contributions to the output uncertainty and risk [11,12]. According to the results of GSA, researchers can reduce the uncertainty of output effectively through allocating more resources (people, time, financial budget, etc.) to the most important input variables and simplify the model by fixing the non-important input variables at nominal values. Due to these advantages, GSA has been widely used in engineering. For instance, Arwade et al. [13] applied GSA on the collapse of a two story two bay frame under gravity load to determine the relative importance of the individual member yield stresses and guide the model reduction. Mukherjee et al. [14] used GSA on a mesoscale model of unreinforced masonry shear wall to find the sensitive parameters. Kala [15,16] applied GSA on steel frame structures to measure the effect of initial imperfections on the load carrying capacity of the structures. Gottvald and Kala [17] used GSA to investigate of the influence of

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the parameters of mining processes on tangential digging forces of the bucket wheel excavator. Hu and Mahadevan [18] proposed an enhanced surrogate model-based reliability analysis method based on GSA. For more details about GSA, one can refer to the reviews about sensitivity analysis [19,20].

In the last several decades, many different GSA methods have been proposed. For instance, screening method [21,22] was proposed for the problems with a lot of input variables and few model assumptions, variance-based method [23–26] was proposed as a quantitative method to detect the variance contribution of input variables to the model output, and moment-independent method [27–29] was proposed to measure the effect of input variables on the whole distribution of model output. In structural reliability analysis, we are more interested in whether the structure fails or not, which can be represented by the sign of model output. Then, the model output can be generally considered as a binary variable. The GSA methods mentioned above mainly focus on the models with real-valued continuous output and could not be used in reliability analysis directly [30]. In traditional reliability sensitivity analysis, the sensitivity is often measured through the partial derivative of failure probability with respect to the distribution parameters [31–34]. These sensitivity indices are local sensitivity measures. Usually, they only measure the effect of input variables on the failure probability at their nominal values. Therefore, a global reliability sensitivity index is required to measure the average effect of input variables on the failure probability in their whole uncertainty ranges and provide importance ranking of input variables. Thus, Cui et al. [35] proposed a failure-probability-based sensitivity index to achieve this purpose. This sensitivity index measures the effect of input variables on the failure probability through the average difference between the unconditional failure probability and conditional failure probability on certain input variable. Thus, it reflects the average changes of the failure probability when the input variable can be fixed. The failure-probability-based sensitivity index has a similar form with the moment-independent based sensitivity index proposed by Borgonovo [28], but focuses on the failure probability which is often related to the tail behavior of the distribution of model output. Later, Lemaître et al. [30] proposed a density-modification-based reliability sensitivity index, which mainly measures how the epistemic uncertainty of input variable affects the failure probability. This sensitivity index is defined through the difference between the original failure probability and the perturbed failure probability obtained by providing a perturbation to an input variable. These two sensitivity indices can be used to measure the effect of input variables on the failure probability effectively. However, they did not provide interaction sensitivity index to measure the interaction effect between different input variables. The interaction effect between different input variables arises when their total effect cannot be represented as the summation of their individual effect. Sometimes, a input variable may affect the model output mainly through interaction effect [15,36,37]. Thus it is important to measure interaction effect between different input variables in structural reliability analysis.

In this paper, a different method is proposed to measure the effect of input variables on the failure of structure, and in particular, to provide the interaction effect of different input variables. This method is mainly based on the idea proposed by Spear and Hornberger [38] and Fenwick et al. [39]. The basic idea is to separate the model output into different classes, then examine the conditional distributions of input variables in each class. If the conditional distributions of a input variable in all classes are similar to each other, then the input variable has little impact on the classification of model output, i.e. the input variable is non-influential to model output. On the contrary, for an influential input variable, the corresponding conditional distributions in all classes are different to each other. There is no assumption on the distribution of input and output variables and the smoothness of the response function. Additionally, only a single set of input–output samples is enough to estimate the sensitivity indices. Initially, Spear and Hornberger [38] separated the model output into two classes and just considered the individual effect of input variables. Later, Fenwick et al. [39] extended the idea of Spear and Hornberger for reservoir models with high-dimensional output. They separated the model output into multivariate classes based on a distance-based criterion [40]. Specially, they proposed an approach to measure the interaction effect of different input variables based conditional distributions.

In structural reliability analysis, the model output can be easily separated into two classes based on whether the structure fails or not. Then, we measure the effect of input variable on the failure of structure through the difference between the original probability density function (PDF) and the failure-conditional PDF of input variable. Similar to the approach used in [39], the interaction effect between two different input variables is measured through the difference between the failure-conditional PDF of a single input variable and the failure-conditional PDF of that input variable additionally conditioned to a second input variable. The rest of this paper is organized as follows. Section 2 introduces the new reliability sensitivity indices. In section 3, the interpretations of the new sensitivity indices are provided. The estimation of the new reliability sensitivity indices is shown in section 4. Several examples are presented in section 5 to show the reasonability of the new reliability sensitivity indices and the feasibility of the proposed estimation method. Section 6 gives the conclusion.

2. The new reliability sensitivity indices

Let $\boldsymbol{X} = (X_1, \ldots, X_d)$ be the d-dimensional vector of random input variables. All the input variables are independent to each other. The PDF of X_i is denoted as $f_{X_i}(x_i)$ $(i = 1, \ldots, d)$ and the joint PDF of $\boldsymbol{X}$ can be represented as $f_{\boldsymbol{X}}(\boldsymbol{x}) = \prod_{i=1}^{d} f_{X_i}(x_i)$. The output variable Y is defined by $Y = g\ (X_1, \ldots, X_d)$, where $g(X_1, \ldots, X_d)$ is the performance function. Let $F = \{g(\boldsymbol{X}) \le 0\}$ denote the failure of structure. Then the failure probability can be defined by $P(F) = P\{g(\boldsymbol{X}) \le 0\}$.

2.1. Individual effect of a single input variable

In structural reliability analysis, the model output can be generally separated into two classes F and $\bar{F}$, where F denotes that the structure fails ($g(\boldsymbol{X}) \le 0$) and $\bar{F}$ denotes that the structure does not fail ($g(\boldsymbol{X}) > 0$). F and $\bar{F}$ are complements to each other. If F is determined, then $\bar{F}$ can also be determined. Now, we examine the difference between the failure-conditional PDF $f_{X_i}(x_i|F)$ and the original PDF $f_{X_i}(x_i)$ of X_i. The difference can be represented through the area closed by these two PDFs, i.e.

$$d_i = \int_{X_i} \left| f_{X_i}(x_i) - f_{X_i}(x_i|F) \right| \mathrm{d}x_i \tag{1}$$

According to the idea used in [38,39], large value of d_i indicates that X_i has significant effect on the failure of structure. Then, the proposed reliability sensitivity index of input variable X_i is defined as

$$S_i = \frac{1}{2} d_i = \frac{1}{2} \int_{X_i} \left| f_{X_i}(x_i) - f_{X_i}(x_i|F) \right| \mathrm{d}x_i \tag{2}$$

The properties of S_i are shown in Table 1.