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Free vibration analysis of deep doubly curved open shells using the Ritz method

K. Malekzadeh Fard [a], A.M. Baghestani [b,*]

[a] Department of Structural Analysis and Simulation, Aerospace Research Institute, MalekAshtar University of Technology, Tehran, Iran
[b] Department of Mechanical Engineering, Babol Noshirvani University of Technology, Shariati Street, Babol, Mazandaran 47148-71167, Iran

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ABSTRACT

This paper develops a unified semi-analytical method for the free vibration analysis of moderately thick doubly curved open shells with arbitrary geometry and classic boundary conditions. The only restriction on the shell's geometry is that boundaries are being coincided by the principal curvature lines of the shell. The formulation is based on the first order shear deformation theory by considering effects of curvature in the evaluation of stress resultants. Differential geometry method is used to represent the arbitrary shape of the middle surface of the shell. The Ritz method with algebraic polynomials as trial functions is employed to obtain the natural frequencies and mode shapes of the shell. To demonstrate the efficiency and accuracy of the solution, convergence and comparison studies are carried out for a cantilevered shallow shell and a parabolic panel. Furthermore, a variety of new vibration results including frequencies and mode shapes of ellipsoid and Enneper panels with various boundary conditions are presented which may be used as benchmark results for future studies.

1. Introduction

Shells are widely used as structural or machine components in various engineering disciplines such as aerospace, civil, marine and mechanical engineering. One of the main features of shells is ability to survive extreme loading which results from their curvature. Shells may have a great variety of geometries and curvatures that make it possible to select an appropriate geometry for a special application. However, most studies about mechanical behavior of shells have been devoted to special shell geometries such as singly-curved or revolution shells. Therefore, developing a method for analyzing shells with general geometry is worthwhile and of great significance. Due to the importance of vibration characteristics such as natural frequencies and mode shapes in proper design of a structure, the present study focuses on the free vibration analysis of a shell with arbitrary geometry. Studies on free vibrations of shells are quite extensive. In an excellent monograph by Leissa [1], researches on the vibration analysis of thin shells before 1970s were reviewed. There are also more recent survey such as Liew et al. [2] and Qatu et al. [3] which review articles about shallow shells and dynamic behavior of composite shells, respectively. Here some selective published articles on free vibration of doubly curved shells are mentioned.

Li et al. [4] used spline finite strip method for free vibration analysis of doubly curved shells that their boundaries coincide with principal curvature lines. The accuracy and versatility of the proposed method were demonstrated by computing natural frequencies of some typical shells. Fan and Luah [5] considered the free vibration analysis of arbitrary thin shell structures by spline finite element method. Natural frequencies of shells with different geometries including spherical, cylindrical and shells of revolution as well as single- and double-cell boxes, were obtained. Kang and Leissa [6] presented three dimensional (3D) free vibration analysis of complete paraboloidal shells of revolution with variable thickness by Ritz method. Numerical results were presented for a variety of paraboloidal shells having uniform or variable thickness, and being either shallow or deep. Furthermore, comparisons were made between frequencies obtained from developed 3D formulation and a 2D thin shell theory. The free vibration of skewed open circular cylindrical deep shells was studied by Kandasamy and Singh [7]. They used a numerical method based on the Ritz method to calculate frequencies and mode shapes of skewed panels clamped at the curved edges and free at the straight parallel edges. Ye et al. [8] presented the free vibration analysis of cylindrical, conical and spherical open isotropic shells subjected to arbitrary boundary conditions. Classical shell theory and Ritz method with Chebyshev polynomials as the trial functions were

* Corresponding author.
E-mail address: am.baghestani@nit.ac.ir (A.M. Baghestani).

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used to solve the problem. They modeled various boundary conditions by introducing artificial distributed linear and rotational springs at each ends of the shell. Numerical results including natural frequencies and mode shapes were presented for open shells with different classical and elastic boundary conditions. Xie et al. [9] developed this method, for the vibration analysis of FGM doubly curved shells of revolution. Ye et al. [10] utilized modified Fourier series method to analyze free vibration of composite laminated doubly-curved shells of revolution. They obtained natural frequencies and mode shapes of circular toroidal, elliptical, paraboloidal and hyperbolical shells for different material parameters and various boundary conditions. Furthermore, vibration analysis of laminated shell structures by accounting general boundary conditions with artificial spring techniques in the Ritz method was carried out by Jin et al. [11]. The free vibration of doubly curved anisotropic laminated composite shells using Hierarchical Trigonometric Ritz Formulation was analyzed by Fazzolari and Banerjee [12]. They used different kinematics descriptions such as equivalent single layer, Zig-Zag and layer-wise theories to study effects of significant parameters on natural frequencies of deep cylindrical, spherical and hyperbolic paraboloidal shells. Fazzolari [13] developed a mixed displacement/transverse stress approach for the free vibration analysis of laminated composite and FGM doubly curved shells. The usefulness of the presented approach lies in the fulfillment a priori of the interlaminar equilibrium of the transverse shear and normal stresses. Thakar and Ray [14] analyzed the effect of thickness coordinate to radius ratio (z/R), on fundamental frequency of three types of laminated doubly curved shells such as cylindrical, spherical and hyperbolic paraboloidal shells by finite element method. Extensive researches on free vibration analysis of shells with various geometries were performed by Tornabene and co-workers [15]. Recently, Tornabene et al. [16] analyzed the free vibration of conical, cylindrical and planar elliptic structures made of laminated composite materials. They utilized several higher order models to obtain natural frequencies of shells by generalized differential quadrature (GDQ) method and comparisons were made by correspondent finite element solution. Tornabene et al. [17] studied the free vibration of arbitrary shaped laminated composite shells which were modeled by means of non-uniform rational b-splines. A mapping technique and GDQ method were utilized for the solution and natural frequencies and mode shapes of different shell geometries were presented. The studies about vibration of shallow shells are vast and a few recent studies are mentioned here. Ye et. al. [18] presented a unified solution method for the free vibration analysis of composite shallow shells with general elastic boundary conditions. Fazollari [19] developed an analytical formulation for free vibration analysis of doubly curved laminated composite shallow shells by combining the dynamic stiffness method and a higher order shear deformation theory. Hirvani et al. [20] developed a model to consider effect of delamination on the vibration behavior of the laminated composite shells. They used finite element method to compute natural frequencies of some shallow shells including hyperboloid and elliptical panels. The literature survey reveals that semi-analytical studies on the free vibration of deep open shells with general configurations are very limited. Therefore, the objective of the present study is filling this gap and developing a unified, efficient and accurate procedure for free vibration analysis of open shells with arbitrary geometry and various boundary conditions.

In this article, free vibration of shells in principal curvilinear coordinate is studied by Ritz method. Differential geometry concepts in conjunction with first order shear deformation theory (FSDT) are utilized to formulate the problem. Strain and kinetic energies of the shell are obtained and then the Ritz method is used to reduce the problem to a standard eigenvalue problem. Algebraic polynomials are selected as admissible functions to compute natural frequencies and mode shapes of shells with various classic boundary conditions. To assess the applicability of the method, verification study is carried out for a cantilevered shallow shell and a parabolic panel. Then, parabolic panel with different combination of clamped, simply supported and free boundary conditions is investigated. Lame parameters and principal radii of curvature of an ellipsoid panel are derived in simple and usable form and employed to compute natural frequencies and mode shapes of the panel. Furthermore, a panel in the form of an Enneper surface is analyzed. The present study concerns isotropic shells, but it can be extended to analyze laminated composite or functionally graded shells easily.

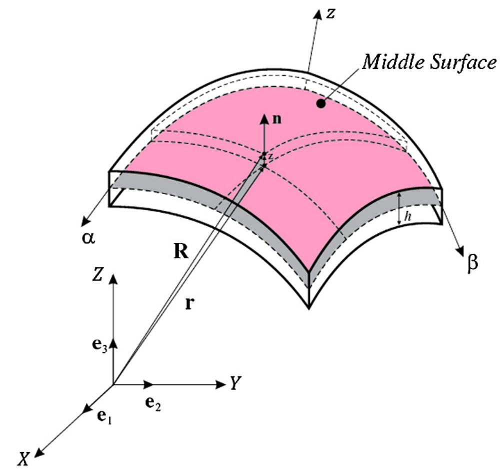

Fig. 1. Doubly curved shell geometry.

2. Formulation

2.1. Kinematic relations and stress resultants

Consider an isotropic doubly curved shell with uniform thickness h, as shown in Fig. 1. The orthogonal curvilinear coordinate system (α, β, z) is chosen such that α and β curves are lines of curvature, that are tangent to a direction of principal curvature, on the mid-surface $(z = 0)$ while axis z is normal to the mid-surface. The boundaries of the shell coincide with the principal axes. Since, the present formulation is developed to analyze shells with general geometry, a systematic approach based on differential geometry concepts is preferable. The middle surface of the shell can be defined in terms of a position vector $\boldsymbol{r}(\alpha, \beta)$ as

$$\boldsymbol{r}(\alpha,\beta) = X(\alpha,\beta)\boldsymbol{e_1} + Y(\alpha,\beta)\boldsymbol{e_2} + Z(\alpha,\beta)\boldsymbol{e_3} \tag{1}$$

where $X(\alpha, \beta)$, $Y(\alpha, \beta)$, $Z(\alpha, \beta)$ are continuous functions of variables α, β and $\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}$ represent unite vectors along global Cartesian coordinate (X, Y, Z), see Fig. 1. Then, an arbitrary point of the shell is

$$\boldsymbol{R}(\alpha,\beta,z) = \boldsymbol{r}(\alpha,\beta) + z\boldsymbol{n}(\alpha,\beta) \quad -h/2 \le z \le h/2 \tag{2}$$

where $\boldsymbol{n}(\alpha, \beta)$ is outward unit normal to the mid-surface. Fundamental parameters of a shell such as the Lame parameters and the principal radii of curvature may be deduced from Eq. (1), [21]. These parameters are very important in mechanical behavior of shells and appear frequently in subsequent equations. The Lame parameters, that pertains to measurement of distances on the surface, can be calculated as