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Parametric studies on elastoplastic buckling of rectangular FGM thin plates

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ABSTRACT

Considering Functionally Graded Material (FGM) plates composed of elastoplastic Ramberg–Osgood metal and elastic ceramic parts, the buckling phenomenon is studied. Plastic flows are formulated by the methods of Incremental Theory (IT) and Deformation Theory (DT). Using proper homogenization tactics and minimum potential energy principle, inclusive forms of governing equations are derived for the elastoplastic buckling of FGM plates. Using the Generalized Differential Quadrature (GDQ) technique a suitable solution algorithm is schemed and programmed in Matlab. Comparing the results with similar studies in buckling of elastoplastic homogeneous plates or elastic FGM plates, the methodology is assessed. The developed analytical IT and DT methodologies are used to analyze the elastoplastic buckling of plates with different boundary conditions. In different in-plane longitudinal and transversal loading conditions, the buckling load of plates with different thickness and aspect ratio are obtained. The effect of other several important factors such as the exponent of power law distribution function and the type of boundary conditions are studied in an extensive range of loads. Between similar FGM rectangular plates, the effect of aspect ratio and thicknesses upon the buckling loads are studied.

1. Introduction

Elastoplastic buckling of plates are extensively occurred in different important civil, mechanical, marine or aerospace structures. Therefore the buckling capacity of the structure to withstand the applied loads prior to collapse is important. Usually in buckling process a plate suddenly jumps from an elastic-compressive regime of behavior-deflections into the elastoplastic compressive-bending regime of behavior-deflections. Especially the phenomenon can be seen in plates with lower yield strength such as aluminum alloys or mild steel plates. The researchers have considered the elastic and elastoplastic buckling of plates by using different analytical [1–5] or numerical approaches [6–9]. They have studied elastoplastic buckling of plates under uniaxial and biaxial loading conditions by using of plastic incremental and total flow rules. For instance, using the incremental theory of plasticity, the elastoplastic buckling of plates and shells under uniaxial and biaxial loads is studied by Grognec [1]. Based on incremental theory, Handelman and Prager [10], Ilyushin [11] and Wang [12] accomplished the plastic buckling analysis. On the other hand, Wang [12], Stowell [13] and Pride and Heimerl [14] studied on the analysis of plastic buckling with total deformation theory. Apparently, the predicted buckling load in incremental and total analyses would be different. Generally the results of total theory are more consistent with experimental results [2,4]. Wang et al. [5] used the Ritz method to study elastic and elastoplastic buckling. Their studies show that the increase of thickness and Ramberg–Osgood material constant deepens the difference between the two plasticity models. Wang and Huang [9] have studied the elastoplastic buckling of rectangular plates under biaxial loadings by using of GDQ method. The outcome of the aforesaid researches confirms better estimations of total theory formulations for the buckling load of thin structures. In this case, similar applications of the incremental theory reveals a kind sensitivity towards the plate dimensions [8,15]. Betten and Shin [16] studied the buckling of a plate with clamped boundary condition under biaxial loadings by using of deformation theory. They concluded that the risk of elastoplastic buckling is increased in thick plates.

Traditional layerwise composites suffer the problem of interfacial stress concentrations and consequently the detachment of laminates. To enhance the load caring capacity of structural materials, in many industrial applications, the use of engineered heterogeneous composites known as Functionally Graded Material (FGM) is growing. Comparing layerwise composites, in FGMs the sharp rise of cross sectional stresses are not seen. These materials have found extensive applications in different purposes such as aerospace in-

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dustries, nuclear reactors, turbines, flywheels or gearing systems. Usually FGMs are made of two metallic and ceramic parts in which the gradation of material properties are continuous and gradual. The reduction in residual and thermal stresses [17] are among the advantages of FGMs. In recent years, the static and dynamic analyses of beams, plates and shells made of FG compounds are extensively represented. In this category, the buckling of plates is among the most important cases. About the nonlinear analysis of FGM plates and shells, some researches are studied and collected by Shen [18]. In this regard Fledman and Aboudi [19] have studied the elastic buckling of FG plate in uniaxial loading conditions. In a similar research, the buckling of thin elastic FG plate is performed by Chu et al. [20] by using the collocation method. Besides, the buckling of FG plates have been analyzed by such researches as Ramu and Mohanty [21] who considered the uniaxial and biaxial compressive loadings conditions. Upadhyay and Shukla [22] dissect on post-buckling behavior of functionally graded skew plates. In the field of elastic buckling of FGM thick plates, one can also point to Reddy [23] and Yang et al. [24].

Despite reach literature in elastic buckling of FGM plates, there are few researches in elastoplastic buckling of FGM plates. For instance, elastoplastic thermal cycling of layered materials was analyzed by Giannakopoulos et al. [25] and elastoplastic crack growth and fracture of FGM compositions has been studied by Jin et al. [26]. More recently, Alijani et al. [27] performed finite element elastoplastic buckling and post-buckling analyses of FGM beams for a bilinear material. Nguyen et al. [28] analyzed elastoplastic buckling of FGM beam by semi inverse method. Up to now there has not been a report on the effect of aspect ratio upon the buckling phenomenon. So in this paper a thorough analysis is provided to study the buckling of elastoplastic FGM plate with different geometrical characteristics.

The metal–ceramic composition of FGM compound is in need of excessive information about the mechanical properties of each constituent and proper method for the mixing and homogenization. One of the well-known methods of homogenization for elastoplastic material is the method known as TTO (after Tamura et al. [29, 30]). Yet using TTO, another similar composition method is introduced by Wilyamson et al. [31], in which two composite phases constructing the FGM compound behave such as a unique isotropic material.

In order to analyze elastoplastic buckling of FGM plates, primarily the applicable governing equations of buckling are developed and represented. Afterwards, the effects of several parameters are considered. The studied parameters include: Material properties of nonlinear elastoplastic FGM compound, the types of boundary conditions, the ratios of biaxial loads, incremental or total models of plastic flow analyses, aspect ratio of the plate and the level of heterogeneity which is reflected in the parameters of volume fraction distribution function.

2. Buckling of elastoplastic FGM plates

In order to introduce the most important geometrical parameters affecting the derivation of elastoplastic FGM plate governing equations, a plate with biaxial loading pattern is depicted in Fig. 1. In Fig. 1, ξ is the ratio of longitudinal compressive traction to the transversal tensile traction applied to opposite sides of the plate. Besides the plate dimensions in x, y and z directions are a, b and h respectively. Considering large deformations and classical deformation fields, non-zero components of total strain in the plate under in-plane loading conditions are extracted as [32],

$$\varepsilon_x = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2}$$

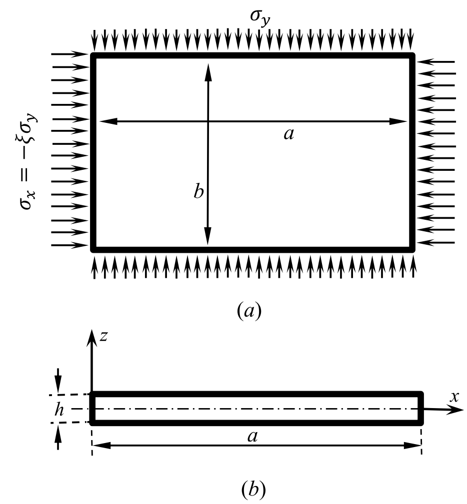

Fig. 1. Geometrical and biaxial loading parameters of elastoplastic heterogeneous plate: (a) Upper view. (b) Front view.

$$\varepsilon_y = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) - 2z\frac{\partial^2 w}{\partial x \partial y} \tag{1}$$

in which u_0, v_0 and w is the mid-plane deflection in x, y and z direction respectively. The principle of minimum total potential energy is used to obtain equilibrium constraints. Here total potential energy is the sum of internal strain energy and the energy conserved by external loads. That is,

$$\Pi = U + V \tag{2}$$

In which internal energy is provided as [33],

$$U = \frac{1}{2}\int_V \{\dot{\sigma_x}\dot{\varepsilon_x} + \dot{\sigma_y}\dot{\varepsilon_y} + \dot{\tau}_{xy}\dot{\gamma}_{xy}\}dV \tag{3}$$

Besides, the external work of in-plane loaded straight tractions is represented by [33],

$$V = \frac{1}{2}\int_A \left\{\left(\int_{-h/2}^{h/2} \dot{\sigma_x}dz\right)\left(\frac{\partial w}{\partial x}\right)^2 + \left(\int_{-h/2}^{h/2} \dot{\sigma_y}dz\right)\left(\frac{\partial w}{\partial y}\right)^2 + 2\left(\int_{-h/2}^{h/2} \dot{\tau}_{xy}dz\right)\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\right\}dA \tag{4}$$

2.1. Local homogenization, global configuration

As usual, in order to model the properties of FGM structures, a description is needed to show the global distribution of material properties. But, at each point the overall material properties must be related to the properties of local constitutive elements. Accordingly in this study using an adaptable power law model, the distribution of volume fraction in terms of plate thickness is assumed to follow,

$$V_1 = \left(\frac{h - 2z}{2h}\right)^n \tag{5}$$

$$V_1 + V_2 = 1 \tag{6}$$

In which V_1 and V_2 are volume fractions of part 1 and part 2 in FGM compound and n is the exponent of the function. In (5),