Aerospace Science and Technology 67 (2017) 193–205

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Three-dimensional cooperative guidance law for multiple missiles with finite-time convergence

Junhong Song [a], Shenmin Song [a,*], Shengli Xu [b]

[a] Center for Control Theory and Guidance Technology, Harbin Institute of Technology, 150001, Harbin, China
[b] Institute of Shanghai Academy of Spaceflight Technology, Shanghai Electro-mechanical Engineering Institute, 201109, Shanghai, China

ARTICLE INFO

Article history:
Received 8 October 2016
Received in revised form 6 February 2017
Accepted 6 April 2017
Available online 10 April 2017

Keywords:
Cooperative guidance law
Impact angle constraints
Finite-time consensus
Adaptive control
Super-twisting sliding mode
Nonsingular fast terminal sliding mode

ABSTRACT

For the three-dimensional terminal guidance problem of multiple missiles cooperatively intercepting a maneuvering target, a novel finite-time cooperative guidance law with impact angle constraints is proposed. It is continuous and requires no information on target maneuvers. Firstly, the multiple missiles cooperative guidance model with impact angle constraints is constructed. We subsequently divide the process of cooperative guidance law design into two stages. In the first stage, based on the adaptive super-twisting algorithm and integral sliding mode, a new finite-time consensus protocol, i.e., the acceleration command on the line-of-sight direction, is designed to guarantee that all missiles reach the maneuvering target simultaneously. In the second stage, a new adaptive nonsingular fast terminal sliding mode control law, i.e., the acceleration command on the normal direction of the line-of-sight, is developed to ensure the finite-time convergence of the line-of-sight angular rate and line-of-sight angle between individual missile and the target. Furthermore, the detailed finite-time stability analysis is given based on the Lyapunov theory. Finally, numerical simulations demonstrate the effectiveness of the proposed cooperative guidance law.

1. Introduction

As the development of missile guidance technology and the progress of anti-missile technology in recent years, multi-layered defense system for missile is being perfected, and then penetration difficulty of traditional single missile is increasing. So, a concept on multiple missiles cooperative attack, which means that multiple missiles attack a single target or multiple targets cooperatively or, in a specific mission, simultaneously [1], is proposed. Multiple missiles cooperative combat can not only enhance the overall combat capability, improve the hit probability, but also complete the task that the traditional single missile cannot accomplish. Therefore, the study of multiple missiles cooperative guidance law has a very important engineering significance and received much attention.

At present, some multiple missiles cooperative guidance laws especially for a static or low-speed target are studied at home and abroad. Impact time control guidance law was introduced for the first time in [1] and applied to salvo attack missions. Based on virtual leader scheme, a new guidance law with time constraints was developed in [2]. The idea of this scheme is to adopt a virtual leader for real missiles to convert a guidance problem with time constraints to a tracking problem, thus the salvo attack missions were finished. In [3], with use of the dynamic surface control theory and disturbance observation technology, an integrated guidance and control law is proposed for multiple missiles attacking targets cooperatively. In [4,5], using the error of the estimated and desired impact time to design sliding mode surface, sliding mode guidance laws with impact time control were designed. In order to increase the effectiveness of warheads against targets and achieve the best destroying effect, a specific impact angle needs to be considered. In [6], a new guidance law to control both impact time and impact angle was proposed, which can be applied for an efficient salvo attack of anti-ship missiles. A new guidance law with adjustable coefficients to control the impact time and impact angle simultaneously is proposed in [7]. Ref. [8] developed a novel sliding mode-based impact time and impact angle guidance law for engaging a modern warfare ship. However, the above-mentioned studies only aimed at a static or low-speed target, the impact time must be set before the attack, and missiles do not exchange information in the process of guidance. Therefore, these approaches are a static guidance strategy, which cannot be called as a true multiple missiles cooperative attack. At present, less research has been paid on the multiple missiles intercepting a single maneuvering target simultaneously [9,10]. So, in this article, we pay more atten-

* Corresponding author.
E-mail address: hitsjh@163.com (S. Song).

http://dx.doi.org/10.1016/j.ast.2017.04.007
1270-9638/

Nomenclature

$r, \dot{r}$	Relative distance and relative velocity between missile and target
$q_\varepsilon, \dot{q}_\varepsilon$	Elevation angle and its rate of the line-of-sight
$q_\beta, \dot{q}_\beta$	Azimuth angle and its rate of the line-of-sight
$a_{Mr}, a_{M\varepsilon}, a_{M\beta}$	Components of missile acceleration along the line-of-sight axes
$a_{Tr}, a_{T\varepsilon}, a_{T\beta}$	Components of target acceleration along the line-of-sight axes
$q_{\varepsilon d}, q_{\beta d}$	Desired terminal elevation and azimuth angle of the line-of-sight
d_q	Term of lumped disturbance on the normal direction of the line-of-sight
d_r	Term of lumped disturbance on the line-of-sight direction
t_{go}	Time-to-go for the missile's terminal guidance
i	Subscript to denote the index of missile
t_{fi}	Predictive engagement instant at time t for the ith missile
Δ_{ri}	Unknown upper bound of the lumped disturbance d_{ri} for the ith missile
Δ_{qi}	Unknown upper bound of the lumped disturbance d_{qi} for the ith missile

tion to the design of cooperative guidance law with impact angle constraints for multiple missiles intercepting a single maneuvering target simultaneously and existing dynamic information sharing in the process of guidance.

An implicit constraint in multiple missiles cooperative interception is that the engagement must be completed in finite time. Cooperative problem imposes a finite-time consensus problem of multi-agent systems. Finite-time consensus protocols were developed by the sign function of states [11,12] or sliding mode control [13] for first-order multi-agent systems with considering disturbance or not. However, all these protocols are discontinuous, which may cause chattering in practice. To reduce the chattering phenomenon, continuous finite-time consensus protocols [14–16] for first-order multi-agent systems have been designed by using Lyapunov finite-time stability. However, these works do not consider the presence of external disturbances in the agents' dynamics. So, continuous finite-time consensus problem in the presence of external disturbances for multi-agent systems remains challenging.

For the terminal guidance problem of the missile intercepting a maneuvering target, finite-time convergence is also an important demand. This can be achieved by a specific kind of guidance law [17–20] based on the terminal sliding mode (TSM) control. However, the proposed guidance laws in [17,18] has a singularity problem that may lead to negative results. In order to resolve the singularity problem, the guidance laws based on the nonsingular terminal sliding mode (NTSM) control are developed in [19, 20]. But, NTSM control has the second problem that it has the slower convergence rate than the traditional linear sliding mode (LSM) control when the system state is far away from the equilibrium. Next, to overcome the two problems simultaneously, some guidance laws based on nonsingular fast terminal sliding mode (NFTSM) control were presented in [21,22]. In addition, super-twisting sliding mode control approach [23–27] is also the well-known approach to realize the finite-time convergence, and the super-twisting sliding mode control approach has been successfully used for the guidance law design [25–27].

In the process of guidance law design, the external disturbances caused by the target acceleration are another important problem about which one should concern. When the disturbances are unknown but bounded and the upper bound on the disturbance is unknown, an efficient approach is adaptive sliding mode control, for the unknown upper bounds of the disturbances can be estimated by adaptive laws. To this end, some adaptive sliding mode guidance laws without information about the bounds of disturbances have been proposed in [17,19–21], which can guarantee the finite-time convergence. However, the above-mentioned adaptive sliding mode guidance laws are all discontinuous for the use of sign function, which leads to the chattering problem inevitably. To alleviate chattering phenomenon, the proposed adaptive sliding mode control laws are modified by using the saturation function to take the place of the sign function in [19–21]. However, the proof of the system stability is not given, and it can only guarantee the convergence to a region, which may cause the parameter drift problem. So, to do this, the disturbance rejection performance is sacrificed to some extent. Another, to solve the parameter drift problem, the adaptive laws can be improved with σ-modification technology [28]. However, the proof of the stability will become difficult.

Motivated by the previous discussion, for multiple missiles intercepting a single maneuvering target simultaneously, a novel three-dimensional cooperative guidance law with impact angle constraints is proposed in this paper. The proposed cooperative guidance law can guarantee that all missiles intercept the maneuvering target with desired impact angle simultaneously. The key point of this guidance law is continuous and fast finite-time convergence. Moreover, it is only required that disturbances are bounded and the bounds can be unknown. The main contributions are summarized as follows:

1) Compared with Refs. [6–8], in this paper the proposed cooperative guidance law is for multiple missiles intercepting a single maneuvering target simultaneously and existing dynamic information sharing in the process of guidance. The designed cooperative guidance law design is divided into two stages. In the first stage, based on the adaptive super-twisting sliding mode control and integral sliding mode, a new finite-time consensus protocol, i.e., the acceleration command on the line-of-sight (LOS) direction, is designed to guarantee that all missiles reach the maneuvering target simultaneously. Thus, the second stage is that a new continuous adaptive NFTSM control law, i.e., the acceleration command on the normal direction of the LOS, is developed to ensure that each missile can intercept the target with the desired impact angle successfully.
2) Different from Refs. [14–16], the designed novel finite-time consensus protocol based on the adaptive super-twisting sliding mode control is continuous and finite-time convergence in the presence of external disturbances for multi-agent systems.
3) Compared with Refs. [17,19–21], the proposed new continuous adaptive NFTSM control law is inherently continuous without using the saturation function to substitute the sign function. In addition, the parameter drift problem is well solved and the finite-time stability is strictly proved which is a great difference between Ref. [28] and this paper.

The rest of the paper is organized as follows. Section 2 introduces some necessary preliminaries and gives the problem formulation of cooperative interception in three-dimensional space.