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# Analytical solution of the optimal steering law for non-ideal solar sail

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ABSTRACT

This paper analyzes the problem of finding the optimal steering law for a flat solar sail whose propulsive acceleration is described by an optical force model. In particular, the problem amounts to looking for the optimal direction of the unit vector normal to the sail plane that maximizes the projection of the propulsive acceleration along a given direction. Starting from the known results from the literature, according to which a close form solution for the general case of not (fully) specularly reflecting sail cannot be retrieved, the propulsive acceleration is approximated by a mathematical model that closely resembles the classical optical force model. Using this new approach, the solution of the optimal steering law is shown to be written in an analytical form that is fully general and extremely accurate. In this sense, the proposed mathematical model may effectively be used to analyze the impact of the thermo-optical characteristics of the sail film on the optimal steering law within a wide range of different mission scenarios.



## 1. Introduction

The solar sail concept represents one of the most promising innovations within the field of low thrust propulsion systems, as is clearly demonstrated by the recent success of the Japanese mission IKAROS [1], when a small solar sail was first deployed and then actively controlled in interplanetary space [2,3]. The renewed impulse received by the photonic-based space propulsion is also confirmed by two new planned NASA missions that are going to be equipped with a solar sail [4,5].

For these reasons, it is extremely useful to have suitable mathematical tools to be used during the mission analysis of a solar sail-based spacecraft. In most cases the solar sail trajectory is analyzed within an optimal framework, by looking for the steering law that maximizes a given scalar performance index, which usually coincides with the total flight time. The literature involving such a subject, starting from the pioneering work by Zhukov and Lebedev [6], offers several examples [7–9] in which the optimal steering law is studied as a function of the physical characteristics of the solar sail reflecting film, that is, of the so called sail force model.

The simplest sail force model, referred to as ideal model, consists in assuming the propulsive acceleration equivalent to that obtained from a flat and specularly reflecting solar sail [10]. The ideal model can be refined by taking into account the thermo-optical characteristics of the reflecting film, but retaining the fundamental assumption of flat solar sail. This corresponds to the optical force model. The sail billowing effect due to the solar radiation pressure is accounted for in the so called parametric force model [11], which relates the thrust vector direction with the sail attitude orientation through an interpolation of numerical–experimental data. However, the optical force model is the best compromise between accuracy and simplicity of the model. In fact, the parametric model is not much used in a preliminary mission analysis for several reasons: it is tailored on a specific sail configuration and requires specific experimental data, it is difficult to update during the mission when, for example, the sail degradation effects must be taken into account, and it is more complex to insert within an optimization algorithm.

The algorithms needed for calculating the optimal steering law in analytic form are extremely important, since they assure a significant reduction of the computational time for simulating the spacecraft optimal trajectories, especially when the solar sail has medium or low performance. Even in the simplified case of a flat solar sail, that is, neglecting any billowing effect, a fully analytical solution of the optimal steering law is available only when the propulsive acceleration is assumed to be directed along the normal to the sail plane. This happens, for example, in the ideal case of fully specular reflection of the impinging photons [7], or in the case of imperfect reflectivity that simply reduces by a factor of $\eta < 1$ the magnitude of the solar radiation pressure force (with respect to the ideal case) without altering its direction [12].

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**Nomenclature**

| | |
|---|---|
| $A$ | function of $\theta$ and $B$, see Eq. (38) |
| $A_s$ | sail area ........ $\mathrm{m}^2$ |
| $\boldsymbol{a}$ | propulsive acceleration ........ $\mathrm{mm/s}^2$ |
| $a_c$ | spacecraft characteristic acceleration ........ $\mathrm{mm/s}^2$ |
| $a_\perp$, $a_\parallel$ | components of the propulsive acceleration along $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{t}}$, respectively ........ $\mathrm{mm/s}^2$ |
| $a_r$, $a_n$ | components of the propulsive acceleration along $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{n}}$, respectively ........ $\mathrm{mm/s}^2$ |
| $B$ | reduced force coefficient |
| $B_b$, $B_f$ | non-Lambertian coefficients of the back and front sail surface |
| $b_1$, $b_2$, $b_3$ | solar sail force coefficients |
| $D$ | function of $\theta$ and $B$, see Eq. (40) |
| $f$ | auxiliary function |
| $J$ | performance index |
| $m$ | spacecraft mass ........ kg |
| $\hat{\boldsymbol{n}}$ | normal unit vector |
| $\mathcal{P}$ | plane spanned by $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{q}}$ |
| $P_\oplus$ | solar radiation pressure at $r = 1$ au ........ $\mathrm{N/m}^2$ |
| $Q$ | polynomial in $x$ |
| $\hat{\boldsymbol{q}}$ | fixed unit vector |
| $\boldsymbol{r}$ | Sun–spacecraft vector, with $r = \lVert\boldsymbol{r}\rVert$ ........ au |
| $r_\oplus$ | reference distance (1 au) |
| $s$ | specular reflection fraction of sail film |
| $\hat{\boldsymbol{t}}$ | unit vector parallel to the sail plane |
| $x$ | auxiliary variable |
| $\alpha$ | cone angle ........ deg |
| $\alpha^\star$ | optimal cone angle ........ deg |
| $\widetilde{\alpha}$ | cone angle corresponding to a local maximum of $J$ ........ deg |
| $\varepsilon$ | error parameter |
| $\epsilon$ | emissivity of sail film |
| $\eta$ | reduction coefficient of the sail thrust in the $\eta$-perfect reflection model |
| $\theta$ | cone angle of $\hat{\boldsymbol{q}}$ ........ deg |
| $\theta_i$ | function of $B$, with $i = 1, 2, 3, 4$ |
| $\rho$ | reflection coefficient of sail film |
| $\phi$ | function of $\theta$ and $B$, see Eq. (39) |

*Subscripts*

| | |
|---|---|
| $a$ | approximated |
| max | maximum |
| min | minimum |
| $b$ | back surface |
| $f$ | front surface |

*Superscripts*

| | |
|---|---|
| $\wedge$ | unit vector |

The simplified force models are still used in optimization problems for mission applications, in order to reduce their complexity and computational costs [13,14]. However, as is thoroughly discussed in Ref. [9], when a more realistic reflection model is used [15–18], as it happens for the optical force model [11], the optimal control law cannot be obtained in a completely analytical form. Even though the approach described in Ref. [9] has been shown, along the years, to be a useful tool for analyzing the optimal performance of a solar sail with an optical force model in various mission scenarios [19], the algorithm proposed in Ref. [9] has some intrinsic limitations. In fact, the determination of the optimal direction of the normal to the sail plane, which must be calculated at each simulation step, requires the use of a root-finding method.

The aim of this work is to start from the exact mathematical model of Ref. [9] and to introduce a suitable approximation to the propulsive acceleration model such that the optimal steering law may be obtained in a completely analytical form. It will be shown that the new solution turns out to be extremely accurate and capable of describing the impact of the thermo-optical characteristics of the sail film on the optimal steering law within a wide range of mission scenarios.

## 2. Solar sail propulsive acceleration model

The propulsive acceleration $\boldsymbol{a}$ of a flat solar sail, at a distance $r$ from the Sun, can be conveniently described by means of the optical force model [9,11,10] as

$$\boldsymbol{a} = \frac{2\,P_\oplus\,A_s}{m}\left(\frac{r_\oplus}{r}\right)^2 (\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{r}})\left[b_1\,\hat{\boldsymbol{r}} + (b_2\,\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{r}} + b_3)\,\hat{\boldsymbol{n}}\right] \quad (1)$$

where $P_\oplus = 4.563\ \mu\mathrm{N/m}^2$ is the solar radiation pressure at a distance $r = r_\oplus \triangleq 1$ au from the Sun, $A_s$ is the sail area, $m$ is the solar sail-based spacecraft total mass, $\hat{\boldsymbol{r}}$ is the Sun–sail (radial) unit vector, and $\hat{\boldsymbol{n}}$ is the unit vector normal to the sail plane in the direction opposite to the Sun, see Fig. 1.

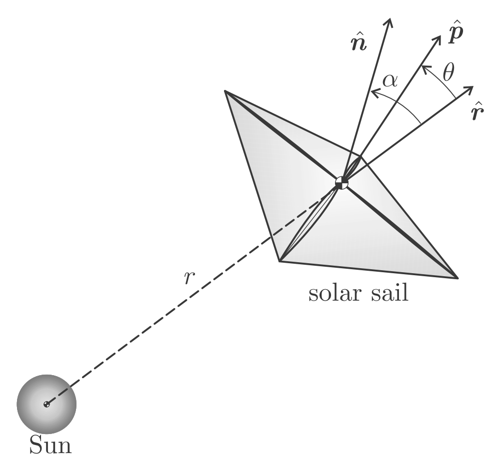


**Fig. 1.** Flat solar sail characteristic angles.

In Eq. (1), $b_1$, $b_2$, and $b_3$ are the force coefficients, defined as

$$b_1 = \frac{1 - \rho\,s}{2} \quad (2)$$

$$b_2 = \rho\,s \quad (3)$$

$$b_3 = \frac{B_f\,\rho\,(1-s)}{2} + \frac{(1-\rho)\,(\epsilon_f\,B_f - \epsilon_b\,B_b)}{2\,(\epsilon_f + \epsilon_b)} \quad (4)$$

where $\rho$ is the reflection coefficient, $s$ is the fraction of photons that are specularly reflected, $B_f$ (or $B_b$) is the non-Lambertian coefficient of the front (or back) sail surface, $\epsilon_f$ (or $\epsilon_b$) the emissivity coefficient of the front (or back) sail surface. Note that in the special case $\hat{\boldsymbol{n}} \equiv \hat{\boldsymbol{r}}$ (i.e., for a Sun-facing sail) and $r = r_\oplus$, the modulus of the propulsive acceleration is usually called spacecraft characteristic acceleration and is referred to as $a_c$. From Eq. (1), it is found that

$$a_c = \frac{2\,P_\oplus\,A_s}{m}\,(b_1 + b_2 + b_3) \quad (5)$$



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