Aerospace Science and Technology 66 (2017) 152–159

Contents lists available at ScienceDirect

Aerospace Science and Technology

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Robust model reference adaptive control based on linear matrix inequality

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ARTICLE INFO

Article history:
Received 30 October 2016
Received in revised form 20 January 2017
Accepted 9 March 2017
Available online 14 March 2017

Keywords:
MRAC
Robust
LMI
Lyapunov function

ABSTRACT

Model reference adaptive control (MRAC) has been used in numerous applications to improve system performance in the presence of system uncertainties. To achieve stringent tracking performance specifications, fast adaptation is required in the MRAC framework. However, fast adaptation with high gain adaptive rates could cause high frequency oscillation in the control response, resulting in system instability. In this paper, a novel method is proposed to improve the transient performance, and to restrain high frequency oscillation of the control signal, without modifying the selected reference model. An error feedback compensator is introduced into the control signal in this method, to restrict the control signal oscillation, caused by the estimate error of unknown parameter. Based on the augmented error dynamics, the compensator is designed as a robust controller. Moreover, the error feedback matrix can be obtained by solving a set of linear matrix inequalities. The control method is applied to a controlled wing rock aircraft dynamics model to verify the effectiveness. Simulation results show that the proposed method allows for fast adaptation with high gain adaptive rates, while eliminating high frequency oscillation and guaranteeing transient performance.

1. Introduction

As a popular control methodology of increasing interest for applications in both science and engineering domains, model reference adaptive control (MRAC) has its unique capabilities to accommodate system parametric, and structural uncertainties caused by payload variations, component failures, and external disturbances [1]. The MRAC was firstly introduced to improve the aircraft performance in the presence of system uncertainties [2,3]. In MRAC, the controller gains were updated based on the error between a selected reference model and the uncertain system, and to make the uncertain system be able to track the reference model. However, the controller performance could be negative affected by the excessive errors, and many efforts were paid to improve the transient behavior of the tracking errors in the last few years. The majority of these efforts led to non-adaptive high gain feedback [4,5], switching control law [6], and a parameter-dependent persistent excitation condition [7]. Another approach to decrease the tracking error magnitude and achieve fast adaption is to increase the adaptive learning rates. However, high gain adaptive learning rates could result in high frequency oscillations in the control signal, which could excite unmolded dynamics [8], and lead to system instability [9,10].

From the perspective of improving the transient performance and restraining the high frequency oscillations, a great deal of effort has been dedicated to modify the control architecture and the adaptive laws. A composite adaptive controller was proposed, which suggested a new adaptation law using both tracking error and prediction error, which led to less oscillatory behavior in the presence of high adaptive rates as compared to MRAC [11–14]. Cao and Hovakimyan [15,16] proposed a L1 adaptive control design method. In this architecture, a low pass filter was employed in the control signal to filter out the high frequency components, so that the L1 adaptive control theory could guarantee transient performance and robustness in the presence of fast adaptation. Yucelen and coworkers [17,18] presented a new framework involving a modification term in the update law, which filtered out the high frequency content contained in the update law while preserving asymptotic stability of the system error dynamics. This key feature allows for robust, fast adaptation with high gain adaptive learning rates. Recently, Annaswamy and coworkers [19–21] analyzed

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http://dx.doi.org/10.1016/j.ast.2017.03.017
1270-9638/

a Luenberger observer-based adaptive control scheme to improve transient performance, where the reference model was modified by adding a state error (or output error) feedback.

In this paper, a robust model reference adaptive control based on linear matrix inequality (RMRAC-LMI) is proposed. The RMRAC-LMI attempts to minimize the effect of the high frequency oscillation caused by the estimation errors by adding an error feedback term in the control signal. A design guideline is provided for the selection of the feedback gain based on the linear matrix inequality. The proposed RMRAC-LMI enables one to achieve close tracking of reference signals and allows for fast adaptation with high gain adaptive rates, while eliminating high frequency oscillation and guaranteeing transient performance.

The remainder of this paper is structured in the following manner. Section 2 provides preliminaries related to the standard MRAC. Section 3 presents the proposed RMRAC-LMI method, analyzes the stability properties for the case of systems with matched uncertainty, and detailed the design procedure of the error feedback matrix. Section 4 presents the simulation results for an illustrative example, which demonstrates the adaptive and robust performance of our controller. Finally, Section 5 concludes this paper.

2. Control problem formulation

Consider an uncertain system given by

$$\dot{x}(t) = Ax(t) + B\big(u(t) + \theta^T\Theta(x(t))\big) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are known matrix, $\theta \in \mathbb{R}^{s\times m}$ is the unknown constant weight matrix, $\Theta(\cdot): \mathbb{R}^n \to \mathbb{R}^s$ is a known vector of basic functions in the form of $\Theta(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_s(x)]^T \in \mathbb{R}^s$. Furthermore, the pair (A, B) is assumed to be controllable, $x(t)$ is available for feedback, and $u(t)$ is the class of admissible controls consisting of measurable functions.

The reference model is described as follows,

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{2}$$

where $x_m(t) \in \mathbb{R}^n$ is the reference state vector, $r(t) \in \mathbb{R}^r$ is a bounded piecewise continuous reference input, $A_m \in \mathbb{R}^{n\times n}$ is Hurwitz, and $B_m \in \mathbb{R}^{n\times r}$ with $r \le m$. Since $r(t)$ is bounded, it follows that x_m is uniformly bounded for all $x_m(0)$.

The underlying problem here is to design a control input u in Eq. (1) so that the closed loop system has bounded solutions and x tends to x_m with bounded errors in the presence of the uncertainties. Consider the following feedback control law,

$$u(t) = u_b(t) + u_{ad}(t) \tag{3}$$

where $u_b(t)$ is a baseline feedback control given by

$$u_b(t) = -K_1 x(t) + K_2 r(t) \tag{4}$$

where $K_1 \in \mathbb{R}^{m\times n}$ and $K_2 \in \mathbb{R}^{m\times r}$ are baseline control gains such that the following equations hold,

$$A_m = A - BK_1, \qquad B_m = BK_2 \tag{5}$$

And $u_{ad}(t)$ is the adaptive feedback control component given by

$$u_{ad}(t) = -\hat{\theta}^T(t)\Theta\big(x(t)\big) \tag{6}$$

where $\hat{\theta}(t) \in \mathbb{R}^{s\times m}$ is an estimate value of θ, satisfying the weight update law:

$$\dot{\hat{\theta}}(t) = -\Gamma\Theta\big(x(t)\big)e^T(t)PB \tag{7}$$

with $\Gamma = \Gamma^T > 0$, $e(t) = x(t) - x_m(t)$ is the model following error and $P = P^T > 0$ is the solution to the algebraic Lyapunov equation

$$A_m^T P + PA_m + Q = 0 \tag{8}$$

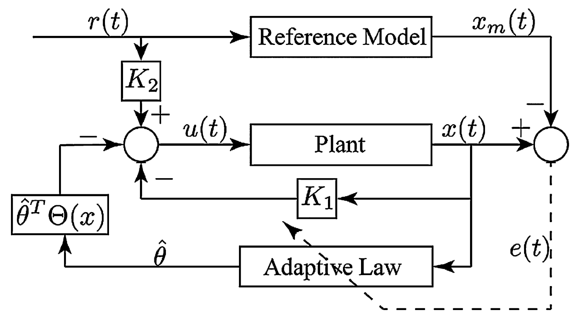

Fig. 1. Architecture of the standard MRAC architecture.

for any $Q = Q^T > 0$. The architecture of the standard model reference adaptive control is given in Fig. 1.

Introducing the parameter estimation error as $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$, the tracking error dynamics can be written as

$$\dot{e}(t) = A_m e(t) + B_m\tilde{\theta}^T(t)\Theta\big(x(t)\big) \tag{9}$$

This control architecture can guarantee asymptotic tracking $x(t) \to x_m(t)$ as $t \to \infty$, while ensuring the boundedness of all closed-loop signals. However, if the adaption rate is increased to obtain better tracking in transient, the transient behavior of $x(t)$ and $u(t)$ cannot be guaranteed because high frequency oscillations are generated in the control signal [22].

3. Robust model reference adaptive control

In this section, a robust model reference adaptive controller based on linear matrix inequality is deduced to restrain the high frequency oscillation caused by the large adaptive learning rates. In the proposed RMRAC-LMI, a state-error feedback is introduced in the control signal. Then, the entirely design procedure of the error feedback matrix is also offered, and its stability properties for the case of systems with matched uncertainty is further proved.

3.1. Robust MRAC architecture

A state-error feedback component is introduced in the standard model reference adaptive control law (Eq. (3)) to form the robust model reference adaptive scheme,

$$u(t) = u_b(t) + u_{ad}(t) + u_m(t) \tag{10}$$

where $u_m(t)$ is the modified control signal, defined as

$$u_m(t) = K_3\big(x(t) - x_m(t)\big) \tag{11}$$

where K_3 is the state-error feedback gain. Hence, the control signal could be expressed as

$$u(t) = -K_1 x(t) + K_2 r(t) - \hat{\theta}^T(t)\Theta\big(x(t)\big) + K_3\big(x(t) - x_m(t)\big) \tag{12}$$

Substitute Eq. (12) into Eq. (1),

$$\begin{aligned}\dot{x}(t) = {} & (A - BK_1)x(t) + BK_2 r(t) + B\tilde{\theta}^T(t)\Theta\big(x(t)\big) \\ & + BK_3\big(x(t) - x_m(t)\big)\end{aligned} \tag{13}$$

According to the matching condition Eq. (5), the uncertain system could be written in the form of

$$\begin{aligned}\dot{x}(t) = {} & A_m x(t) + B_m r(t) + B\tilde{\theta}^T(t)\Theta\big(x(t)\big) \\ & + BK_3\big(x(t) - x_m(t)\big)\end{aligned} \tag{14}$$

Subtracting Eq. (2) from Eq. (14),

$$\dot{e}_m(t) = A_m e_m(t) + B\tilde{\theta}^T(t)\Theta\big(x(t)\big) + BK_3 e_m(t) \tag{15}$$