Aerospace Science and Technology ••• (••••) •••–•••

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Traffic system operation optimization incorporating buffer size

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ARTICLE INFO

Article history:
Received 21 June 2016
Received in revised form 4 January 2017
Accepted 7 March 2017
Available online xxxx

Keywords:
Air transportation
Air traffic control
Mathematical models
Buffer size
Discrete event system

ABSTRACT

This paper presents a general framework for resolving resource utilizations conflicts of air traffic system, expressed in the form of a max-plus linear model. The general air traffic system optimization model is presented taking into account buffer size in view of different purposes. The proposed air traffic system model is very flexible and it is extensible focusing on distinct circumstances. The dynamics of air traffic systems is characterized through the occurrence of discrete events such as aircrafts entering or leaving sub-segments. We focus on input control of air traffic system modeled by max-plus algebra by controlling the inflow of aircraft into the system. The constraints between input variables, state variables and output variables were obtained. The proposed method aims to minimize system total delay to meet demand specifications, allowing the air traffic controller to obtain the best control policy, which delays the occurrence of input events or varying input rate of system resources. Through simulations we verify the performance of the proposed max-plus linear model control scheme.

1. Introduction

The steady growth of air traffic all over the world leads to complicated air traffic system situations where the aircrafts may infringe safety separation standards in airspace with dense traffic flow. Air traffic management can provide effective decision tool for airspace users according to the preferred flight profiles. Specifically, current approach consists of two main activities aimed at increasing airspace capacity and decreasing air traffic controller workloads, that is, air traffic control and air traffic flow management. Additionally, the increase of air traffic volume urges to improve the efficiency of air traffic system, which was attested by the Next Generation Air Transportation System (NGATS) and the Single European Sky Air Traffic Research System (SESAR) aimed at highlighting the importance of trajectory generation in achieving the targets of increasing capacity and safety while improving flight efficiency [31,38]. The conflict detection and resolution has been a major topic in air traffic management for decades. Future position of aircraft in the airspace can be obtained using aircraft state information available such as position and altitude, as well as trend vector so that potential flight conflicts can be given by predefining metrics. The metric usually include a sole parameter such as distance or a combination of several parameters. The term 'trajectory optimization' is referred as actions taken when the aircraft departure time is known or even after the flight is airborne but with sufficient time to allow a collaborative decision-making process [32]. This term excludes instructions and clearances that require an immediate response.

Over the past years, various optimization models for resolving flight conflict have been put forward. Typically, these methods can be categorized into nominal model, worst case model and probabilistic model [23]. For the nominal method, current aircraft states are projected directly into future [21]. The worst case assumes that an aircraft will perform many kinds of maneuvers. If any one of these maneuvers cause a flight conflict, then a potential conflict is predicted [11]. For the probabilistic method, uncertainties are modeled to describe trajectory uncertainties [17,36]. In addition, aircrafts can use a set of optimization maneuvers stem from optimal control model, heuristics model or multi-agent model in order to solve a flight conflict. Specifically, the most realistic models are developed in the theoretical framework of optimal control [4,7,12,17,20,22,26,35,36,41–43]. As a consequence, the continuous-time nature of the flight conflict resolution problem is conserved, but analytical solutions can be found only for the special case. Another interesting approach for en-route flight conflict resolution is heuristic algorithms [2,27,39,40]. The genetic optimization algorithm is developed based on genetic encoding, where crossover and mutation are introduced in each generation.

Further developments are presented combining speed and altitude changes using mixed-integer linear optimization model [1,19,30]. The logical constraints are used to denote the configuration requirements for maneuvers and the collision avoidance problems

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http://dx.doi.org/10.1016/j.ast.2017.03.012
1270-9638/

Please cite this article in press as: Y.-x. Han et al., Traffic system operation optimization incorporating buffer size, Aerosp. Sci. Technol. (2017), http://dx.doi.org/10.1016/j.ast.2017.03.012

can be combined into a single mixed-integer linear optimization model. Remaining optimization methods for flight conflict resolution rely on game theory [4,8,11,40]. This approach translates safety specifications into restrictions on the system state reachable sets and the Hamilton–Jacobi equations whose solutions describe the boundaries of reachable sets can be acquired using game theory. In other contexts, air traffic management system could also be supposed as a multi-agent system in which different aircrafts working together in order to keep safe. Consequently, distributed artificial intelligence methods and multi-agent system technologies for flight conflict resolution are also put forward. The presented methods involve a set of intelligent agents aiming at achieving some specific goals through negotiation with each other [5,18,24,25,34,37].

High performance flight conflict planning methodologies can increase both airspace efficiency and safety by enforcing the separation activities between aircrafts, making it possible to integrate strategic approach into air traffic system instead of tactical process [33]. Thus, there is an urgent need to develop a novel strategic planning method for air traffic system in order to utilize the air traffic system resource efficiently. To achieve this, air traffic system operation is planned at different time frames [9,10]. Strategic planning is done several days before take-off and consists of assigning flight plans for a whole day. Our work focuses on air traffic system strategic planning. In particular, we studied the max-plus linear systems with time-invariant transition structures. For this class of systems, we build a multiple aircraft trajectory optimization model based on the dynamic of air traffic flow passing through the airspace network.

The outline of this paper is as follows. Firstly, the max-plus theory is introduced and air traffic system single jet route max-plus model with finite buffer size is presented in section 2. Next, air traffic system general linear state-space representation in max-plus algebra is put forward. Section 3 considers the constraints of air traffic system with multiple jet routes. The multi-aircraft trajectory optimization cost function and case studies are demonstrated in section 4 and section 5, respectively. Finally, section 6 gives some conclusions.

2. Modeling of the single jet route with finite buffer size

In general, discrete event systems that model only synchronization aspects are called max-plus-linear systems consisting of man-made systems that contain a finite number of resources, which are shared by several users all of which contribute to the achievement of some common goals [15]. Air traffic system is a typical example of discrete event system essentially consists of air traffic controller, aircraft and airspace network, which changes due to the occurrence of discrete events such as air traffic controllers' instructions so that its behavior is governed by the progression clock ticks. The techniques available for conventional linear systems cannot be extended directly to discrete event systems. This constitutes the motivation for the work reported in this paper [28]. It is well known that max-plus theory play a key role among the modeling methodologies for discrete event system, since they are able to capture the precedence relations and interactions among the concurrent and asynchronous events that are typical in discrete event system. The linear properties of max-plus models that describe air traffic flow make control policies for the airspace very attractive. Attempts like this have been successfully made in many areas such as queuing system, manufacturing systems, telecommunication networks and railway networks [3,6,16,29]. In this paper, we will develop a max-plus-linear framework with finite buffer size for air traffic system and the proposed model can be used to control all resources of the airspace.

Air traffic system is a complicated dynamic system. In particular, the terminology buffer refers to en-route holding patterns or holding procedures notified in relation to an aerodrome. In addition, there are also some connections between buffer and segment or sector capacity for the air traffic system. It is well known that flight levels are used to ensure safe vertical separation between aircrafts, despite natural local variations in atmospheric air pressure. Flight levels solve this problem by defining altitudes based on a standard air pressure at sea-level. All aircrafts operating on flight levels calibrate to this setting regardless of the actual sea level pressure. The number of available flight levels corresponding to holding patterns or holding procedures is called finite buffer size, which will inevitably influence the aircraft trajectory optimization as indicated below. In the following section, we will present the modeling approach focusing on air traffic system with finite buffer size by max-plus algebra theory.

We firstly give the basic definition of the max-plus algebra and present some results on a class of $(\max,+)$ functions. We adopt the convention that for all $x \in R$, $\max(x,-\infty) = \max(-\infty,x) = x$ and $x+(-\infty) = -\infty + x = -\infty$. The two basic operations in the max-plus algebra can be represented by $\oplus$ and $\otimes$, respectively. In particular, ε is the neutral element for the operation $\oplus$ and absorbing for $\otimes$, that is, for all $x \in R_\varepsilon$, $x \otimes \varepsilon = \varepsilon$, $\varepsilon = -\infty$, $R_\varepsilon = R \cup \{\varepsilon\}$. The identity element for the max-plus algebra is $e = 0$. Accordingly, the null matrix $\boldsymbol{\Phi}$ and identity matrix $\boldsymbol{E}$ are defined as: $\phi_{ij} = \varepsilon$ $(i = 1,2,\dots,n;\ \ j = 1,2,\dots,n)$ and $E_{ij} = \begin{cases} 0, & i = j \\ \varepsilon, & i \neq j \end{cases}$ $(i = 1,2,\dots,n; j = 1,2,\dots,n)$, respectively. Furthermore, the basic operations addition $(\oplus)$ and multiplication $(\otimes)$ are defined as follows [15]:

$$x \oplus y = \max(x, y) \tag{1}$$

$$x \otimes y = x + y \tag{2}$$

for numbers $x, y \in R_\varepsilon$ and we extend the $(\max,+)$ algebra operations to matrices in the following way:

$$[\boldsymbol{A} \oplus \boldsymbol{B}]_{ij} = a_{ij} \oplus b_{ij} = \max(a_{ij}, b_{ij}) \tag{3}$$

$$[\boldsymbol{A} \otimes \boldsymbol{C}]_{ij} = \bigoplus_{k=1}^{n} a_{ik} \otimes c_{kj} = \max_{k=1,2,\dots,n} (a_{ik} + c_{kj}) \tag{4}$$

for matrices $\boldsymbol{A}, \boldsymbol{B} \in R_\varepsilon^{m\times n}$ and $\boldsymbol{C} \in R_\varepsilon^{n\times p}$.

We now show by an example how the single jet route with finite buffer size, characterized only by synchronization, can be modeled using max-plus algebra. To obtain the max-plus system model of air traffic system, we firstly decide the characteristic locations through which flight flow rates need to be determined. These characteristic locations consist of air route split or intersection point and airspace fixes. In general, aircraft climb profiles are defined in terms of a constant calibrated airspeed (CAS) segment and a constant Mach segment. Consider a serial air traffic system of multiple sub-segments with finite buffer sizes, aircrafts have to pass through the serial characteristic locations consecutively so as to receive service at each sub-segment. We now consider, for the sake of simplicity, a typical CAS/Mach climbing/descending jet route with finite buffer size as illustrated, for example, by $AB/A'B'$ in Fig. 1. At a coarse grain, we could consider each sub-segment as a 'machine' on which to process multiple tasks with different process time (depending on the aircraft property).

Let n be the number of sub-segment $M_1, M_2, \cdots, M_n$ and m the number of aircraft $P_1, P_2, \dots, P_m$, respectively. In addition, $P_1, P_2, \dots, P_m$ also represents an external arrival stream of aircrafts. Each sub-segment of the jet route has m aircrafts allocated

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