Aerospace Science and Technology 66 (2017) 304–314

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Stochastic analysis of fuel consumption in aircraft cruise subject to along-track wind uncertainty

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ARTICLE INFO

Article history:
Received 20 October 2016
Received in revised form 6 March 2017
Accepted 17 March 2017
Available online 22 March 2017

Keywords:
Stochastic trajectory prediction
Fuel consumption uncertainty
Wind uncertainty
Probability Transformation Method

ABSTRACT

The effects of along-track wind uncertainty on aircraft fuel consumption are analyzed. The case of cruise flight subject to an average constant wind is considered. The average wind is modeled as a random variable, which in this paper is assumed to follow either a uniform or a beta distribution. The probability density function (pdf) of the fuel consumption is obtained using a numerical approach that is based on the Probability Transformation Method (a method that evolves the wind pdf). The dynamics of aircraft mass evolution in cruise flight is defined by a simple nonlinear equation that can be solved analytically; this exact solution is used to assess the accuracy of the method. A general analysis is performed for arbitrary along-track winds. Comparison of the numerical results with the exact analytical solution shows an excellent agreement in all cases. A linear approximation is analyzed as well, which turns out to be very accurate for this problem. The results show that the standard deviation of the fuel mass distribution varies almost linearly with the standard deviation of the wind, whereas the mean of the fuel mass is practically independent of the wind uncertainty. They also show that, for the same along-track wind uncertainty, the uncertainty in the fuel consumption is larger in the case of headwinds than in the case of tailwinds.

1. Introduction

The future Air Traffic Management (ATM) system must address the performance challenges posed by today's airspace: the capacity and the efficiency of the system must be increased while preserving or augmenting the safety levels. To accomplish these goals it is required a paradigm shift in operations through innovative technology and research. In this future system the trajectory becomes the fundamental element of a new set of operating procedures, collectively referred to as Trajectory-Based Operations (TBO), which aim at establishing a trajectory-based ATM system designed to accommodate airspace users' requests to the maximum extent possible [1].

One key factor that affects those challenges is uncertainty, which is an inherent property of real-world socio-technical complex systems, and ATM is clearly not an exception. Uncertainty is critical from different perspectives in air transport: safety, environmental and cost dimensions. Researchers must accept the fact that uncertainty is unavoidable and must be dealt with, rather than ignored. If the capacity of the ATM system is to be increased while maintaining high safety standards and improving the overall performance, uncertainty levels must be reduced and new strategies to deal with the remaining uncertainty must be found. In particular, procedures to integrate uncertainty information into the ATM planning process must be developed. In Rivas and Vazquez [2] one can find a review of all the uncertainty sources that affect the ATM system. Among those, weather has perhaps the greatest impact. Its importance has been extensively assessed in the literature; thus, according to Zelinski and Jastrzebski [3] convective weather is identified as one of the ATM uncertainty factors that most seriously affect the network route structure, and thus the optimal flight trajectory planning.

The analysis of weather uncertainty has been addressed by many authors, using different methods. For instance, Nilim et al. [4] consider a trajectory-based air traffic management scenario to minimize delays under weather uncertainty, where the weather processes are modeled as stationary Markov chains. Pepper et al. [5] present a method, based on Bayesian decision networks, for taking into account uncertain weather information in air traffic flow management. Clarke et al. [6] develop a methodology to study airspace capacity in the presence of weather uncertainty and formulates a stochastic dynamic programming algorithm for traffic flow management. Kim et al. [7] consider different sources of uncertainty (wind and severe weather among them) and derive

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http://dx.doi.org/10.1016/j.ast.2017.03.027
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Nomenclature

A, B	constants of the problem in the aircraft mass equation
C_D, C_L	drag and lift coefficients
C_{D_0}, C_{D_2}	coefficients of the drag polar
c	specific fuel consumption
D	aerodynamic drag
$\mathrm{E}[.]$	expectation
f_y	probability density function of random variable y
g	gravity acceleration
g_A, g_F	wind–mass and wind–fuel transformations
h	altitude
L	lift
m, m_f	aircraft mass and final aircraft mass
m_F	fuel load
n	number of points for the PTM numerical approach
S	wing surface area
T	thrust
t	time
V	aircraft true airspeed
w	average along-track wind speed
$\bar{w}, w_M, w_m$	mean, maximum and minimum values of the average wind distribution
x	horizontal distance
x_f	range
α, β	coefficients of the beta distribution
δ_w	half-width of the wind distribution
ρ	air density
$\sigma[.]$	standard deviation
ϕ	sensitivity function of the aircraft mass with respect to the wind speed

service time distributions for different flight phases, assessing traffic flow efficiency by means of queuing network models. Zheng and Zhao [8] develop a statistical model of wind uncertainties and apply it to stochastic trajectory prediction in the case of straight, level flight trajectories. The importance of weather uncertainty information in probabilistic air traffic flow management is shown in Steiner et al. [10], where the translation of ensemble weather forecasts into probabilistic air traffic capacity impact is described. An analysis of wind-optimal cruise trajectories using ensemble probabilistic forecasts and a robust optimal control methodology is performed in Gonzalez-Arribas et al. [9].

In this paper a probabilistic analysis of aircraft fuel consumption taking into account wind uncertainty is presented. Several methods have been proposed to study uncertainty propagation in dynamical systems. The easiest, but more expensive in computational terms, is the classical Monte-Carlo method. Halder and Bhattacharya [11] classify those methods in two categories: parametric (in which one evolves the statistical moments) and non-parametric (in which the probability density function is evolved). In this work, a non-parametric method is applied, in which the wind probability density function (pdf) is evolved. The method used for the uncertainty propagation is based on the Probabilistic Transformation Method (see Kadry [12] and Kadry and Smaily [13]). This method was presented in Vazquez and Rivas [14] where the propagation of uncertainty in the initial aircraft mass was studied, and some preliminary results applied to wind uncertainty are described in Vazquez and Rivas [15]. The approach is based on the resolution of the variational equation for the sensitivity function with respect to the wind.

In this paper a general analysis is performed focusing the study on the cruise phase. This study is relevant because wind is one of the main sources of uncertainty in trajectory prediction, and because cruise uncertainties have a large impact on the overall flight since the cruise phase is the largest portion of the flight (at least for long-haul routes). In particular it is expected that this study be relevant for the determination of the contingency fuel, and, hence, for allowing a more effective decision making, as concluded by SESAR WP-E IMET project (http://www.sesarju.eu/print/2352). In this respect, Hao et al. [16] analyze the cost of carrying the additional discretionary fuel excessively loaded, above a reasonable and conservative buffer, to face flight unpredictability; and Ryerson et al. [17] stress the possibility of achieving substantial savings through the use of a reformed policy for discretionary fuel loading.

The evolution of the aircraft mass in cruise flight is described by a simple nonlinear equation that can be solved analytically. Hence, this exact solution represents a benchmark that is used to assess the performance of the proposed numerical method. The comparison shows an excellent agreement. Moreover, a linear approximation is also made, which is shown to be very accurate for the problem considered in this paper (being within 2% of the exact solution).

In this work, results are presented for arbitrary winds that follow continuous uniform and beta distributions (both symmetric and non-symmetric beta distributions). The uncertainty of the aircraft mass is analyzed first, and its evolution along the trajectory is described. Then, the effects of wind uncertainty on fuel consumption are studied. The results show that the uncertainty in the fuel consumption is larger in the case of headwinds than in the case of tailwinds, for the same value of the wind uncertainty. With respect to the fuel mass distribution, it is shown that the mean is practically independent of the wind uncertainty, whereas its standard deviation does depend on the wind uncertainty, dependence that is roughly linear (for the range of wind uncertainty considered in the paper). These trends are hinted by the results given by the linear approximation.

The outline of the paper is as follows: first, the problem of cruise fuel consumption subject to uncertain winds is formulated (Section 2); in Section 3 the probabilistic wind models are described; in Section 4 an analysis of aircraft mass uncertainty is presented, which is followed by an analysis of fuel consumption uncertainty in Section 5; the linear approximation is presented in Section 6; the results are discussed in Section 7; and finally some conclusions are drawn in Section 8.

2. Cruise trajectory subject to uncertain along-track winds

As already indicated, in this paper the fuel consumption in cruise flight is studied. The cruise is supposed to be formed by a given number of cruise segments, each one of them defined by a constant heading, and flown at constant speed and constant altitude, as required by Air Traffic Control (ATC) procedures. In each cruise segment the flight is assumed to be subject to a constant average wind, which can be different for the different segments, thus modeling the along-track wind variation; the average wind in each segment is modeled as a random variable.

In this paper, as a first step in this research, the case of a cruise defined by only one segment is considered. The case of several cruise segments is left for future work since it involves more than one random variable.

To study the evolution of the aircraft mass in cruise flight, the equations of flight mechanics for flight in a vertical plane (constant course) are considered, under the following hypothesis: symmetric flight, flat Earth model, constant altitude, and constant speed.