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Thermomechanical buckling and post-buckling of cylindrical shell with functionally graded coatings and reinforced by stringers

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ABSTRACT

The cylindrical shells reinforced by stringers have been widely used in modern engineering structures such as storage tanks, missile, submarine hull, oil-transmitting pipeline, etc. In this present article, the thermomechanical buckling and post-buckling behaviors of a cylindrical shell with functionally graded (FG) coatings are investigated by an analytical approach. The cylindrical shell is reinforced by outside stringers under torsional load in the thermal environment. The layers of FG coatings are assumed to be made by functionally graded materials (FGMS) combining of ceramic and metal phases and the core of the shell is made from homogeneous material. The classical shell theory based on the von-Karman assumptions is used to model the thin cylindrical shell. Using Galerkin's procedure and Airy stress function, the governing equations can be solved to obtain the closed-form solution for the critical buckling load and postbuckling load-deflection curves of simply supported shells. Moreover, many important parametric studies of stringers, temperature field, material volume fraction index, the thickness of metal layer, etc. are taken into investigation. According to numerical examples, it is revealed that the outside strings have considerably impact on thermomechanical buckling and postbuckling behaviors of the shells.

1. Introduction

Due to the development of the materials science, a new type of composite materials known as functionally graded materials (FGMs) has been widely used in various engineering applications, especially in high temperature status. FGMs are produced by a continuous mixture of metal and ceramic in which material properties vary smoothly and continuously in the preferred, generally in the thickness direction [1,2]. The metal gives high roughness while the ceramic provides high temperature-resistance and high corrosion-resistant. The concept of FGM was first introduced by a group of material scientists in Japan in the mid-1980s [3,4]. Since then, many investigations, due to their fascinating features of potential applications, have been carried out to study the mechanical buckling, thermal buckling, and stability of the structures made of FGM.

The cylindrical shell structures, as a type of fundamental structural components, are widely in the engineering applications such as a missile, submarine hull, oil-transmitting pipeline, aircraft, hydrospace, shipbuilding construction [5–11]. For example, the strategic missiles using solid materials, they are capable to fly far beyond the continent with great velocity, so their hull could stand very high strength and high temperatures. To satisfy it, the shell of the strategic missiles is usually made of composite carbon–carbon or FGMs. Furthermore, the functionally graded cylindrical shells could also be used as the shell of a nuclear reactor or special engineering pipes [12]. In practice, this kind of structures is often subjected to the intricate environment and complex loading conditions. Thus, it is of great importance to achieve an insight into thermomechanical buckling behavior of the FG cylindrical shell for the engineers and designers.

In the research fields of cylindrical shells, many studies have been addressed for the mechanical buckling and postbuckling problems as Shen [13–16], Huang et al. [17–20], Sofiyev [21], Dai and Zheng [22], Dai et al. [23], Bagherizadeh et al. [24], Akbari Alashti and Ahmadi [25], Sun et al. [26], Duc et al. [27,28], Thang et al. [29]. Many investigations on analytical and numerical methods have been conducted to analyze the thermal buckling behavior

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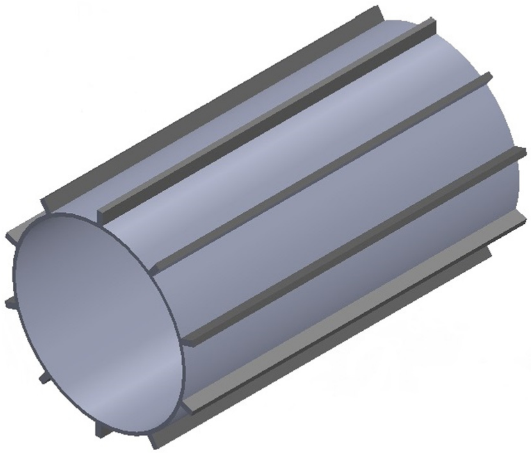

Fig. 1. Configuration of circular cylindrical shell with FG coatings and reinforced by stringers.

of the FG cylindrical shell such as Shahsiah and Eslami [30], Shen [31], Wu et al. [32], Yaghoobi et al. [33], and Thang [34]. An effective meshfree method with highly smoothing shape functions for buckling analysis of Kirchhoff–Love cylindrical was investigated by Wang et al. [35].

For the torsional buckling problem, Li et al. [36] considered a cylindrical crack located in an FGM interlayer between two coaxial elastic dissimilar homogeneous cylinders and subjected to a torsional impact loading. Singh et al. [37] studied the torsional vibrations of functionally graded finite cylinders. Arghavan and Hematiyan [38] presented an analytical formulation for torsional analysis of functionally graded hollow tubes of arbitrary shape. Wang et al. [39] proposed an analytical solution for transient torsional responses of a finitely long, functionally graded hollow cylinder. The torsional impact problem of a cylindrical interface crack between an FGM interlayer and its external homogeneous cylinder was investigated by Feng [40]. Shen [41] also investigated the torsional buckling and postbuckling of FGM cylindrical shells. Najafov et al. [42] examined the torsional vibration and stability problems of FG orthotropic cylindrical shells in the elastic medium. Huang [43] investigated the buckling behaviors of elasto-plastic functionally graded cylindrical shells subjected to torsional load.

Recently, the torsional vibration and buckling analysis of cylindrical shell with FG coatings has been investigated by Sofiyev and Kuruoglu [44]. In this study, the thermomechanical buckling and post-buckling behaviors of cylindrical shells with FG coatings are investigated by analytical approach. The highlight of this study is that the cylindrical shell is reinforced by the outside stringers. Moreover, the cylindrical shells with FG coatings are subjected to torsional load in the thermal environment. The layers of FG coatings are assumed to be made by functionally graded materials (FGMS) combining of ceramic and metal phases and the core of the shell is made from homogeneous material. By using the Galerkin procedure, the closed form expressions can be determined to obtain the critical buckling load and post-buckling response. Several numerical examples are also presented to demonstrate the nonlinear behaviors of cylindrical shell with FG coatings.

For clarity, the content of this research paper is organized as follows: Section 2 presents the configuration of the cylindrical shell with FG coatings and the outside stringers system. Mathematical modeling of the cylindrical shell with FG coatings is analyzed in Section 3. Sections 4 presents the thermomechanical buckling and post-buckling analysis. Several numerical examples are given in Section 5. Finally, the concluding remarks are drawn in Section 6.

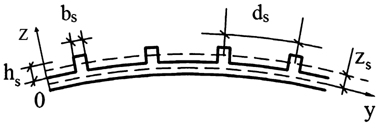

Fig. 2. Coordinate system of axial stiffeners.

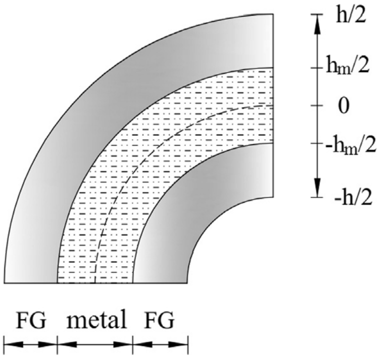

Fig. 3. Configuration for cylindrical shell with FG coatings.

2. Configuration of the cylindrical shell with FG coatings and the stringers

Consider a circular cylindrical shell with FG coatings with mean radius R, length L, and thickness h. The cylindrical shell is reinforced by the outside stringers system as shown in Fig. 1 and Fig. 2. Following the power-law distribution in z-direction, the volume fraction of the metal component, V_m can be determined as [29] (see Fig. 3)

$$V_m(z)=\begin{cases}\left[\frac{-2|z|+h}{(h-h_m)}\right]^{\xi}, & -\frac{h}{2}\le z\le-\frac{h_m}{2} \text{ or } \frac{h_m}{2}\le z\le\frac{h}{2},\\ 1, & -\frac{h_m}{2}\le z\le\frac{h_m}{2},\end{cases} \tag{1}$$

where ξ is the volume fraction exponent which dictates the material variation profile through the thickness of the shell. The non-homogeneous material properties of cylindrical shell with FG coating are obtained by the rule of mixture as follows:

$$P=P_{mc}V_c(z)+P_c \tag{2}$$

in which $P_{mc}=P_m-P_c$ and P_m, P_c are the thermomechanical properties of the metal and ceramic, respectively. In order to accurately analysis the thermomechanical characteristics of the shell, the temperature dependency of material constituents is taken into account as [31]

$$P(T)=P_0\big(P_{-1}T^{-1}+1+P_1T^1+P_2T^2+P_3T^3\big), \tag{3}$$

where P_0, P_{-1}, P_1, P_2, and P_3 are the coefficients of temperature T and unique to the constituent materials, and T_0 is the room temperature.

3. Mathematical modeling

According to the classical shell theory with Kirchhoff assumptions, the displacement components of a shell is written as [45,46]

$$\begin{aligned}U(x,y,z)&=u(x,y)-z\frac{\partial w(x,y)}{\partial x},\\ U(x,y,z)&=v(x,y)-z\frac{\partial w(x,y)}{\partial y},\\ W(x,y,z)&=w(x,y),\end{aligned} \tag{4}$$

where (u,v,w) are the displacement components along the (x,y,z) coordinates directions, respectively, of a point on the mid-plane (i.e., $z=0$).