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Pilot induced oscillation suppression controller design via nonlinear optimal output regulation method

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ABSTRACT

This paper proposes a controller design method for preventing pilot-induced-oscillation (PIO) based on nonlinear optimal output regulation theory and center stable manifold method. The type of PIO considered is due to actuator rate limiting and the proposed controller assures C^* handling quality by the optimal control when actuator works in the linear region. The simulation result shows that the proposed nonlinear controller has better tracking performance in comparison with the linear optimal output regulation controller. The robustness of the designed controller against parametrization error, noise and time-delay is also verified.

1. Introduction

Actuator position/rate-limit is an important issue in the design of flight control systems since it is one of the main causes of PIO and is listed in the Category II PIO which are mainly characterized by nonlinearity of actuator [1–3]. Taking a look back at the history of aircraft, some fatal accidents caused by actuator rate-limit were witnessed, such as the crashes of YF-22 in 1992, JAS 39 in 1989, and it even happened in the high-end Shuttle Orbiter during its fifth free flight test in 1977 [4–6]. In all of the aforementioned accidents, the actuator rate-limit happened and caused the degradation in stability and control performance, leading to the strong oscillation in both longitudinal and lateral motions, which ended up to become catastrophes in the first two cases.

Many researches have been conducted in an effort to predict and prevent Category II PIOs. There are several methods for PIO prediction such as OLOP criterion [7], power spectral density of pilot input signals for PIO detection [8], probabilistic neural network method [9], wavelet-based techniques [10] and some other methods which are discussed and evaluated in [2]. The authors in [11] proposed a nonlinear filter to compensate the phase of the control signal before going to the actuator. Since high pilot gain at critical conditions together with actuator rate-limit is a major cause of Category II PIOs, the authors in [12] proposed a solution of designing a reference filter to limit the control amplitude from pilot when PIOs are about to happen. This helps preventing PIOs to some extent and reduces the danger of PIOs [13]. The authors in [14–17] incorporated the constraints in the controller design process, in which the linear control law is rescaled based on the PIO predictions. A drawback of this approach is that the stability is not always guaranteed, and there is a trade-off between control performance and stability guarantees.

Another solution is to design an anti-windup controller or anti-windup-like controller, which is so-called phase compensator [18–21]. One of the most noticeable advantages of these controllers is that the nominal control performance is preserved. When the actuator rate-limit occurs, it causes a phase-lag to the response of the system. This mismatch between unsaturated and saturated signals degrades the handling and flying quality of the aircraft and potential instability rises. The phase compensator has the effect to eliminate the phase shift due to the actuator rate-limit. However, this approach does not consider the stability as well as the performance of the whole nonlinear closed-loop system. The authors of [22] proposed a nonlinear anti-windup for manual flight control. It is admitted that the proposed controller can perform more aggressive maneuvers than the command limiting approach. Also, the stability of the aircraft is guaranteed. In [23], the comparison between phase compensator and an $H\infty$ anti-windup controller is presented. The simulation result shows that the latter provides the better control performance. One more approach to design an anti-windup controller for an experimental aircraft ATTAS is suggested in [24–27]. The feasibility of using an anti-windup controller to en-

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Table 1
Nominal flight conditions.

Parameter	Value	Unit
Altitude h	0	m
Velocity V	153.0	m/s
Angle of attack α	0.03691	rad
Pitch angle θ	0.03691	rad
Pitch rate q	0	rad/s
Elevator angle δ_e	−0.7588	deg
Throttle δ_T	0.1385	–

counter the rate-limit problem is illustrated in both simulation and experiment.

In this paper, we present a nonlinear optimal control design method based on the center stable manifold theory [28] to prevent the PIO phenomenon due to rate-limit while maintaining the performance of the flight control guaranteed by the optimal control. The center-stable manifold approach, which solves the optimal output regulation problem [29–31], is a generalization of the stable manifold method for optimal stabilization (see, e.g., [32–37] for the detail of the stable manifold method). In this approach it is possible to take full account of the rate-limit nonlinearity in the framework of optimal control. We focus only on short period, longitudinal dynamics of the aircraft and the actuator rate-limit is (approximately) modeled as a nonlinear element. The simulation result shows that the proposed controller is not only able to deal with the nonlinearities caused by the rate-limit of actuator, but it is also robust to certain kinds of modeling error, noise and time-delay.

The organization of this paper is as follows. Section 2 describes the mathematical model of the aircraft with consideration of rate-limit. Section 3 presents the description of the PIO suppression problem. In section 4, the nonlinear optimal output regulation controller is designed. Section 5 shows simulation results and discussions. Section 6 concludes this paper. Appendices at the end summarize the computational theory of center stable manifold and nonlinear output regulation.

2. Mathematical model with rate-limit

This section describes the mathematical model of the aircraft including the pilot and actuator model with rate-limit. In general, the aircraft motion model is expressed in six-degree-of-freedom equations of motion as in [38] (page 183). However, since this research considers the PIO problem due to the rate-limit of elevator, only short period approximation for longitudinal motion model is used for designing the controller. In [38] (page 293), a nominal flight conditions of F-16 aircraft is shown in Table 1 and its short period approximation is written as follows

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = A_{\mathrm{sp}} \begin{bmatrix} \alpha \\ q \end{bmatrix} + B_{\mathrm{sp}} \delta_e,$$

where q is the pitch rate defined in the body-axes, α is the angle of attack, δ_e is the deflection of elevator, and the matrices A_{sp}, B_{sp} are

$$A_{\mathrm{sp}} = \begin{bmatrix} -1.01885 & 0.90506 \\ 0.82225 & -1.07741 \end{bmatrix}, \quad B_{\mathrm{sp}} = \begin{bmatrix} -0.00215 \\ -0.17555 \end{bmatrix}.$$

The dynamics of elevator is modeled as first-order lag system with time constant of $T_a = \frac{1}{20.2}$ (s), or equivalently, cutoff frequency $\omega_a = 20.2$ (rad/s) [38]. The actuator model including the rate-limit is represented in Fig. 1. In that figure, ω_a is the cutoff frequency, u' is the control input to the elevator actuator, u_s is the deflection of the elevator δ_e, and the constraints of the actuator are as follows:

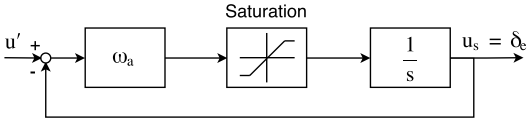

Fig. 1. Actuator model including rate-limit.

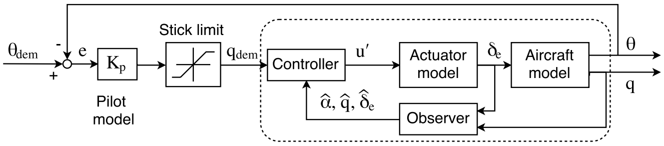

Fig. 2. Control block diagram in longitudinal motion.

- Upper position-limit $\overline{u}$ is 25 (deg), lower position-limit $\underline{u}$ is −25 (deg).
- Upper rate-limit $\overline{u_r}$ is 60 (deg/s), lower rate-limit $\underline{u_r}$ is −60 (deg/s).

In the controller design process, only actuator rate-limit is considered. However, in the nonlinear simulation, both actuator rate-limit and position-limit are taken into account. The approximate actuator model with rate-limit (Fig. 1) can be written in state-space representation as

$$\begin{aligned} \dot{u}_s &= \mathrm{sat}(\omega_a(u' - u_s); \overline{u_r}, \underline{u_r}) \\ &= -\omega_a u_s + \mathrm{sat}(\omega_a u'; \overline{u_r} + \omega_a u_s, \underline{u_r} + \omega_a u_s). \end{aligned}$$

In the above equations, the notation "sat" denotes the saturation function which is defined as

$$\mathrm{sat}(Y; \overline{Y}, \underline{Y}) := \begin{cases} \overline{Y} & \text{if } \overline{Y} \leqslant Y \\ Y & \text{if } \underline{Y} \leqslant Y \leqslant \overline{Y} \\ \underline{Y} & \text{if } Y \leqslant \underline{Y}, \end{cases}$$

where Y is an argument with the upper bound $\overline{Y}$, lower bound $\underline{Y}$ and $\underline{Y} < 0 < \overline{Y}$.

Fig. 2 shows the control block diagram of the aircraft. The augmented system composed of aircraft motion model and actuator model is written as follows

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} A_{\mathrm{sp}} & B_{\mathrm{sp}}\omega_a^{-1} \\ 0_{1\times 2} & -\omega_a \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ x_u \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_a \end{bmatrix} \mathrm{sat}(u; \overline{u_r} + x_u, \underline{u_r} + x_u), \qquad (1)$$

where $x_u = \omega_a \delta_e$, $u = \omega_a u'$. Note that, u is the control input of the augmented system (1) while u' is the real input signal to the actuator model. Since our goal is prevention of PIO, a simple pilot model is considered [14,39,40], which is a pure proportional feedback of the error between the pitch angle of the aircraft θ and the pitch attitude angle command θ_{dem} and the output of pilot model is the desired pitch rate q_{dem}

$$q_{\mathrm{dem}} = K_p(\theta_{\mathrm{dem}} - \theta), \qquad (2)$$

where K_p is the pilot model gain. The value of K_p will be determined in Section 5.

3. Description of the PIO suppression problem

In the previous section, the augmented system (the one inside the dashed-line box in Fig. 2) including aircraft motion and actuator models is described in (1). We consider q_{dem}, the output of