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A new adaptive finite time nonlinear guidance law to intercept maneuvering targets

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ABSTRACT

A new adaptive nonlinear guidance law for homing missiles to intercept maneuvering targets in terminal phase is proposed. This guidance law generates smooth acceleration commands and is able to stabilize the relative lateral velocity in a desired finite time. The proposed guidance law uses bounds of the target acceleration and jerk and is consisting of two adaptive terms. It is proved that the first adaptive term of the proposed guidance law converges to the target acceleration normal to the line of sight. Finite time stability of the guidance loop is proved by using Lyapunov stability theorem. Numerical simulations are performed to illustrate the proposed guidance law potential.

1. Introduction

Proportional navigation (PN) and its variants have been widely used as homing guidance laws because they are highly efficient and easy for implementation. The PN guidance law has the required accuracy to intercept a non-maneuvering target or a weakly maneuvering target. However, for the task of intercepting a target with maneuverability close to that of a missile, PN guidance laws are unable to achieve the required precision [1–3].

Recently to deal with maneuverable targets, control theories have been used to design of guidance laws. Lyapunov-based nonlinear guidance law [4] and H_∞ guidance laws [5–7] were designed. But these are not guidance laws with finite time convergence.

In many applications, the time of termination is really quite short. For example, in the space interception where a missile is intercepting a ballistic target, sometimes the time of terminal guidance is only several seconds such that the guidance law is required to ensure finite time convergence of the line of sight angular rate [3]. The problem of finite-time stabilization for nonlinear systems is studied in some references such as [8–10]. Also, finite time guidance laws are proposed in [11–17]. These guidance schemes are consist of a non-smooth sign function that causes chattering in guidance command. Due to chattering, the application of these guidance laws usually is not possible in guidance loop, since the profile generated by the guidance system must be followed by the autopilot in control loop [18]. Therefore to apply these guidance laws, some modifications are required. Usually a saturation function is used in lieu of the sign function for the purpose of removing the chattering. However, this method brings a finite steady state error and lead to tracking within a guaranteed precision rather than perfect tracking [19–21].

The other way to eliminate chattering is to use second order sliding mode (SOSM) control. Guidance laws based on the second-order sliding-mode are designed in [18,22–24]. For the SOSMs, commonly the finite time convergence analysis has been done without the use of the Lyapunov function approach in uncertain nonlinear systems. Some approaches depend on the state derivatives on, so-called, homogeneity principle [18,25], which also does not allow to estimate the reaching time. Some studies are based on disturbance observer [23,26] and [27]. The main drawbacks in these guidance laws are the absence of a formal closed-loop system stability proof and also existence of sign function.

Recently, adaptive nonlinear controllers have been proposed, the interest being the adaptation of uncertainty effects. Then, the control gain is reduced, therefore the control signal has lower chattering [19,24]. Some adaptive sliding mode control schemes are proposed in the literatures [20,28–30]. In [28] only asymptotic stability is guaranteed and the main drawback of the approaches that are proposed in [20] and [29], is the chattering which is not completely eliminated. Finally in [30] the finite time convergence of the closed loop system dynamics and adaptive variable is not ensured.

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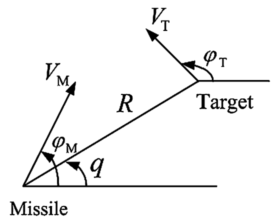

Fig. 1. Missile–target engagement geometry.

The current paper presents a new simple adaptive nonlinear guidance law with finite time convergence. The obtained guidance law generates smooth acceleration commands. Also, the finite time stabilization of guidance loop is proved by using Lyapunov method. The main advantage of the proposed algorithm is availability of the convergence time relation.

The paper is organized as follows. In Sec. 2 the equation of motion is formulated. In Sec. 3, the proposed adaptive nonlinear guidance law is designed. Numerical simulation results are shown in Sec. 4, and conclusions are reported in Sec. 5.

2. Equations of motion

In this section, the model of guidance loop dynamical system is introduced. Consider a two-dimensional interceptor and target engagement as shown in Fig. 1.

It is assumed that the missile and the target are point masses moving in plane. Then, the missile–target engagement model shown in Fig. 1 can be described by the following nonlinear differential equations [17,18] and [23]:

$$\begin{aligned}
\dot{R} &= V_R \\
\dot{V}_R &= \frac{V_q^2}{R} + A_{T,R} - \sin(q-\phi_M)A_M \\
\dot{q} &= \frac{V_q}{R} \\
\dot{V}_q &= -\frac{V_q V_R}{R} + A_{T,q} - \cos(q-\phi_M)A_M
\end{aligned} \tag{1}$$

where R is the relative range, q is the LOS angle, $\dot{q}$ is the line of sight rate, $V_q = R\dot{q}$ is the relative lateral velocity, ϕ_M and ϕ_T represent the flight path angles of the target and missile, respectively, $A_{T,R}$, $A_{T,q}$ are projections of bounded target acceleration along and orthogonal to LOS and A_M is the interceptor acceleration. The object of guidance law is nullify the relative lateral velocity ($V_q = R\dot{q} \to 0$) in a finite time. This will lead to a collision with the target [11].

3. Adaptive nonlinear guidance law

In this section the proposed adaptive nonlinear guidance law is introduced. Assume that the target acceleration normal to LOS $A_{T,q}$ is uncertain with $|A_{T,q}| \le L_{A_T}$ and its first derivative bounded with $|\dot{A}_{T,q}| \le L_{\dot{A}_T}$.

Theorem. *The following guidance law*

$$\begin{cases}
A_M = \dfrac{1}{\cos(q-\phi_M)}\left(-\dfrac{V_q V_R}{R} + k_1 V_q |V_q|^{-\gamma} + k_2\xi_1 + k_3\xi_2\right) \\
\dot{\xi}_1 = \dfrac{V_q}{|V_q|} \\
\dot{\xi}_2 = k_2 k_3 |\xi_2| \dfrac{V_q}{|V_q|} - k_4 \dfrac{\xi_2}{|\xi_2|}
\end{cases} \tag{2}$$

with conditions

$$\begin{cases}
k_1, k_2 > 0 \\
k_4 = L_{\dot{A}_T}\left(L_{A_T} + k_2|\xi_1|\right) + \eta \\
\eta = \text{Positive Constant} \\
0 < \gamma < 1
\end{cases} \tag{3}$$

nullifies the relative lateral velocity in a finite time.

Proof. Substituting guidance law (2) in the relative equation of motion (1) gives

$$\begin{cases}
\dot{V}_q = A_{T,q} - k_1 V_q |V_q|^{-\gamma} - k_2\xi_1 - k_3\xi_2 \\
\dot{\xi}_1 = \dfrac{V_q}{|V_q|} \\
\dot{\xi}_2 = k_2 k_3 |\xi_2| \dfrac{V_q}{|V_q|} - k_4 \dfrac{\xi_2}{|\xi_2|}
\end{cases} \tag{4}$$

Now, consider the following Lyapunov function candidate:

$$V = k_2|V_q| + \frac{1}{2}(A_{T,q} - k_2\xi_1)^2 + |\xi_2| \tag{5}$$

which is positive definite with $k_2 > 0$. By taking the time derivative of V when $V > 0$, using (4), we obtain:

$$\begin{aligned}
\dot{V} &= k_2\frac{V_q}{|V_q|}\dot{V}_q + (A_{T,q} - k_2\xi_1)(\dot{A}_{T,q} - k_2\dot{\xi}_1) + \frac{\xi_2}{|\xi_2|}\dot{\xi}_2 \\
&= k_2\frac{V_q}{|V_q|}\left(A_{T,q} - k_1 V_q|V_q|^{-\gamma} - k_2\xi_1 - k_3\xi_2\right) \\
&\quad + (A_{T,q} - k_2\xi_1)\left(\dot{A}_{T,q} - k_2\frac{V_q}{|V_q|}\right) \\
&\quad + \frac{\xi_2}{|\xi_2|}\left(k_2k_3|\xi_2|\frac{V_q}{|V_q|} - k_4\frac{\xi_2}{|\xi_2|}\right) \\
&= k_2\frac{V_q}{|V_q|}(A_{T,q} - k_2\xi_1) - k_2\frac{V_q}{|V_q|}k_1 V_q|V_q|^{-\gamma} \\
&\quad - k_2\frac{V_q}{|V_q|}k_3\xi_2 + \dot{A}_{T,q}(A_{T,q} - k_2\xi_1) \\
&\quad - k_2\frac{V_q}{|V_q|}(A_{T,q} - k_2\xi_1) + k_2k_3|\xi_2|\frac{V_q}{|V_q|}\frac{\xi_2}{|\xi_2|} - k_4\frac{\xi_2}{|\xi_2|}\frac{\xi_2}{|\xi_2|} \\
&= -k_1k_2|V_q|^{1-\gamma} + \dot{A}_{T,q}(A_{T,q} - k_2\xi_1) - k_4
\end{aligned} \tag{6}$$

By choosing $k_1, k_2 > 0$ and $k_4 = L_{\dot{A}_T}(L_{A_T} + k_2|\xi_1|) + \eta$, where η is a strictly positive constant, yields:

$$\begin{aligned}
\dot{V} &= -k_1k_2|V_q|^{1-\gamma} + \dot{A}_{T,q}(A_{T,q} - k_2\xi_1) \\
&\quad - L_{\dot{A}_T}\left(L_{A_T} + k_2|\xi_1|\right) - \eta \le -\eta
\end{aligned} \tag{7}$$

The condition (7) implies

$$\begin{aligned}
&\int_{V(0)}^{0} dV \le \int_0^t -\eta dt \\
&-V(0) \le -\eta t \\
&t \le \frac{V(0)}{\eta}
\end{aligned} \tag{8}$$

Therefore condition (7) guarantees the convergence of V from $V(0) = k_2|V_q(0)| + \frac{1}{2}(A_{T,q}(0) - k_2\xi_1(0))^2 + |\xi_2(0)|$ to zero in finite time (8). Hence, the convergence of relative lateral velocity to zero and the adaptive term $k_2\xi_1$ to $A_{T,q}$ in the guidance loop dynamics (4) are guaranteed.

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