Aerospace Science and Technology ••• (••••) •••–•••

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Aerospace Science and Technology

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Equilibrium configuration of a bounded inextensible membrane subject to solar radiation pressure

Bo Fu, Rida T. Farouki, Fidelis O. Eke

Department of Mechanical and Aerospace Engineering, University of California, Davis, CA 95616, United States

ARTICLE INFO

Article history:
Received 12 April 2017
Received in revised form 13 June 2017
Accepted 13 June 2017
Available online xxxx

Keywords:
Solar sails
Inextensible membranes
Developable surfaces
Elliptic integrals
Numerical quadrature
Boundary value problem

ABSTRACT

The equilibrium shape of a thin inextensible membrane subject to solar radiation pressure under given boundary constraints is studied. The membrane is assumed to be insusceptible to elastic deformation and to have negligible bending resistance, and its steady-state shape is therefore described by a developable surface (i.e., a surface of zero Gaussian curvature), resulting from an equilibrium between radiation pressure and membrane tension forces. A quantitative understanding of the mechanics of such membranes is essential in characterizing the dynamics of solar sail spacecraft that use sail wing tip displacement as an attitude control mode. The analysis in this paper develops a theoretical foundation for the billowed wing shape. Under reasonable simplifying assumptions, the key result is that solar radiation pressure and a given wing tip displacement yield a billowed solar sail wing with the shape of a generalized cylinder (i.e., a developable ruled surface, whose rulings are all parallel, rather than a general developable with variable ruling directions). The base curve geometry for the solar sail is also determined as the solution to a boundary value problem. The results presented herein allow the shape of the billowed membrane to be computed to any desired precision, for any given tip displacement.

1. Introduction

Solar sails differ from traditional spacecraft in that they harvest momentum from solar radiation pressure, rather than expulsion of onboard propellants. This property of solar sails can significantly reduce mass-to-orbit, especially for round-trip missions requiring a spacecraft to carry return fuel on launch. This results is greatly improved mission cost effectiveness, making solar sails promising candidates for future interplanetary space transportation [15].

Although interest in solar sail technology has grown within the last decade, following the successful demonstration of solar sail technology by the Japanese IKAROS [19] mission in 2010 and also the recent Lightsail-1 mission by the Planetary Society in 2015, the technology is still in a rudimentary state of development. Most solar sail studies have thus far focused on small sails. However, the idea of large solar sails is not new [2–4,9,13,21]. Large sails have the potential to carry payloads up to several metric tons, and could be the key to cost-effective space cargo transportation.

Attitude control remains a critical area of interest in the exploitation of solar sail technology. This is because attitude control for solar sails requires methodologies fundamentally different from those employed by traditional spacecraft. A deployed solar sail will require continuous attitude correction, due to the misalignment of the sail center of pressure (cp) and the sail center of mass (cm). The cm–cp misalignment results from both manufacturing and deployment imperfections, and thus cannot be avoided. The large moment of inertia of a deployed solar sail will also require large body moments for attitude maneuvers, limiting the use of traditional attitude control methods. The reader may consult the paper [20] by Wie for a more detailed discussion of the limitations of traditional attitude control methods in solar sails.

For a solar sail, it makes sense to use solar radiation pressure (SRP) as the *only* source for attitude control moment generation. This can be done by manipulating the cm and cp positions on the sail. Most attitude control methods proposed for solar sails require the addition of substantial mass to the craft as the sail size increases. Recently, an attitude control methodology was proposed by Fu and Eke [7] that does not require significant additional mass. This approach employs a square solar sail, as shown in Fig. 1.

The square sail consists of four right triangular wings, connected to the support booms only at their tips and at the center. For example, in Fig. 1 wing A is attached to the support booms at points O, P, and Q. The basic idea is to displace the sail membrane-boom attachment point, such as point P, a distance δ toward the center O, so that the membrane billows under solar radiation pressure. This billowing of the sail membrane causes a

E-mail addresses: bofu@ucdavis.edu (B. Fu), farouki@ucdavis.edu (R.T. Farouki), foeke@ucdavis.edu (F.O. Eke).

http://dx.doi.org/10.1016/j.ast.2017.06.018
1270-9638/

Please cite this article in press as: B. Fu et al., Equilibrium configuration of a bounded inextensible membrane subject to solar radiation pressure, Aerosp. Sci. Technol. (2017), http://dx.doi.org/10.1016/j.ast.2017.06.018

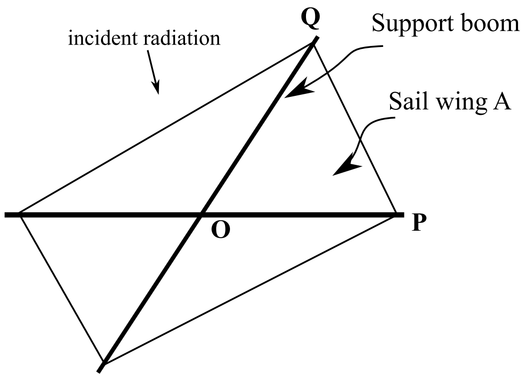

Fig. 1. Schematics of square solar sail.

shift in both the membrane cm and cp, which in turn induces a body torque that can be exploited for attitude control. Simultaneous displacements of more than one of the four wing tips in a square solar sail allow body moments to be generated about all three body axis directions. The reader may consult [7] for a more detailed description of the method, which will be referred to as the Tip Displacement Method (TDM) of attitude control.

An accurate estimate of the shape of the billowed sail wing under a given tip displacement is critical in the effective application of TDM in solar sail attitude control, since the SRP forces and torques are a direct result of the billowed shape. Several authors [10,11,17,22] have proposed methods to estimate the deformed shapes and resulting total forces on solar sails, using a variety of approaches. The analysis in [7] was based upon some reasonable assumptions concerning the shape of the deformed membrane, to facilitate estimation of the resulting attitude torques. The main assumption is that the shape of the billowed wing is a portion of a right circular cylinder. A more rigorous study, that does not invoke the right circular cylinder assumption, was conducted by Fu and Eke [8]. By means of a numerical optimization method, it was found that the billowed wing shape differs from a cylindrical shape, but for small tip displacements this deviation is small and a cylindrical shape is sufficient for attitude torque estimations.

Although the studies [7] and [8] extended knowledge of the TDM and billowed wing profile, a sound theoretical basis for understanding the billowed wing shape was lacking. In [7], the shape was assumed to be cylindrical, and in reference [8], the shape was assumed to be a generalized cylinder, with the base curve obtained numerically via an optimization algorithm. This paper provides a deeper understanding of the wing profile through a first-principles approach to the mechanics of bounded inextensible membranes, and a rigorous method of solving the boundary value problem that determines the equilibrium shape of the sail wing base curve, for any given wing tip displacement. The sail wing is modeled as an inextensible membrane, whose billowed shape is determined by equilibration of solar radiation pressure and internal membrane stresses, upon imposing membrane boundary conditions — namely, the displacement of one wing tip.

Although many aspects of plate and membrane mechanics have previously been studied in depth [1,5,6,12,14,16], the authors are unaware of any prior detailed investigation of the specific problem addressed herein, concerning the equilibrium shape of an initially flat membrane that is inextensible but offers no bending resistance, and billows under the action of uniform pressure when one tip is displaced. The deformed shape of an inextensible membrane must be a *developable*, i.e., a ruled surface of zero Gaussian curvature [18]. A developable surface may be a *cylinder* (whose rulings are all parallel); a *cone* (whose rulings all pass through a fixed point); or a *tangent developable* (the surface generated by the family of tangent lines to a given space curve — the most general case).

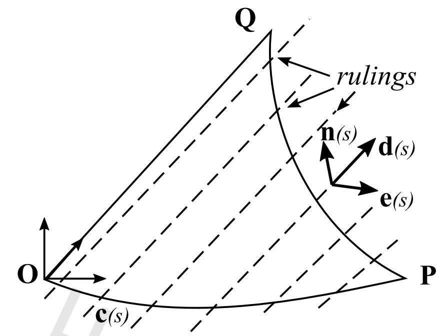

Fig. 2. Billowed sail wing.

Through a detailed investigation of the force and torque equilibrium of a differential membrane surface element, it is shown in this paper that the billowed sail necessarily assumes the shape of a (generalized) cylinder, i.e., the surface generated by a family of parallel lines emanating from a given "base" curve.

2. Kinematics of a solar sail wing

As noted above, a deformed inextensible membrane assumes the shape of a developable surface, a special type of ruled surface that can be *developed* [18] or "flattened" by pure bending action, without any stretching/compressing. The development process preserves distances, angles, and surface areas.

Consider a billowed inextensible solar sail wing, as shown in Fig. 2. If the sail boundary OP is described by a "base curve" $\mathbf{c}(s)$, the sail surface admits a parameterization of the form

$$\mathbf{r}(s,t) = \mathbf{c}(s) + t\,\mathbf{d}(s)\,, \tag{1}$$

where $\mathbf{d}(s)$ is a unit vector specifying the ruling direction at each point of $\mathbf{c}(s)$ and t is distance along the ruling originating from $\mathbf{c}(s)$. Inextensibility of the curve $\mathbf{c}(s)$ is enforced by assuming an arc-length parameterization, i.e.,

$$|\mathbf{c}'(s)| \equiv 1. \tag{2}$$

Note that arc-length parameterization is common in describing inextensible objects, such as ropes or chains.

Expression (1) describes a general ruled surface, which is not necessarily developable. To define a developable surface, appropriate to the context of an inextensible solar sail membrane, the arc-length parameterization condition (2) is not sufficient, and we must impose another condition which ensures that $\mathbf{r}(s,t)$ has zero Gaussian curvature. The Gaussian curvature of a parametric surface can be determined [18] as follows (for brevity, we henceforth omit the dependence of $\mathbf{c}$ and $\mathbf{d}$ and their derivatives $\mathbf{c}', \mathbf{c}''$ and $\mathbf{d}', \mathbf{d}''$ on s). The first partial derivatives of $\mathbf{r}(s,t)$ are

$$\mathbf{r}_s = \mathbf{c}' + t\,\mathbf{d}'\,, \qquad \mathbf{r}_t = \mathbf{d}\,,$$

and, if they are linearly independent, the unit surface normal is specified by

$$\mathbf{n} = \frac{\mathbf{r}_s \times \mathbf{r}_t}{|\mathbf{r}_s \times \mathbf{r}_t|} = \frac{(\mathbf{c}' + t\mathbf{d}') \times \mathbf{d}}{|(\mathbf{c}' + t\,\mathbf{d}') \times \mathbf{d}|}\,.$$

Now since $|\mathbf{d}| = 1$ and $\mathbf{d} \cdot \mathbf{d}' = 0$, the first fundamental form of $\mathbf{r}(s,t)$ has the coefficients

$$E := \mathbf{r}_s \cdot \mathbf{r}_s = |\mathbf{c}' + t\,\mathbf{d}'|^2\,, \quad F := \mathbf{r}_s \cdot \mathbf{r}_t = \mathbf{c}' \cdot \mathbf{d}\,, \quad G := \mathbf{r}_t \cdot \mathbf{r}_t = 1\,.$$

Similarly, the second partial derivatives of $\mathbf{r}(s,t)$ are

$$\mathbf{r}_{ss} = \mathbf{c}'' + t\,\mathbf{d}''\,, \quad \mathbf{r}_{st} = \mathbf{d}'\,, \quad \mathbf{r}_{tt} = \mathbf{0}\,,$$

and the second fundamental form has the coefficients

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