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An accurate and efficient computational method for structural dynamic stresses under random loading

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ABSTRACT

One of challenging issues in structural random analysis is to accurately predict the dynamic stresses. It is even important for fatigue life prediction and strength-safe design. This paper proposes an accurate and efficient computational method to obtain the dynamic stresses of structures under random loading. Firstly, the random loading given by PSD is transferred into harmonic functions. Meanwhile, structural modal stress analysis is conducted. Based on the modal stress superposition and the equivalent treatment for higher modal responses, the dynamic stresses are determined. It consists of two parts: one is from the modal stress superposition, and the other is from the equivalent analysis including the contributions of the ignored higher modes. This new method is verified by the experiment conducted in our laboratory, showing that the dynamic stresses predicted by the new method agree well with the experimental results, and it is of high computational efficiency. The proposed method can be easily applied to dynamic stresses prediction for engineering structures under random loading.

1. Introduction

Random vibration analysis is one of common tasks in designing aeronautic and airspace structures [1]. When a structure encounters random loading, especially if that random excitation is large or its band covers several natural frequencies of the structure, dynamic stresses are usually large resulting in vibratory fatigue failure [2,3]. For aircrafts, vibration fatigue failure usually happens at the area with a protrusion, such as ventral fin, or local resonant areas with flaws or stress concentration, and it would seriously affect their service and safety. In evaluating the strength and the fatigue-life of structures, precise prediction on dynamic stresses is of great importance.

For given excitations, the system equation of motion describes structural dynamic behavior. Responses, like displacements, velocities, accelerations, strains and stresses, can be obtained by solving this mathematical equation [4,5]. For deterministic loading, there are many methods to solve the equation in time domain, such as immediate integration method, mode superposition method, mode acceleration method [6–10]. Among them, the modal displacements are usually superposed, however the modal stresses are not handled frequently by structure analysts. For random loading, the correlation functions of responses are the convolutions of the correlation functions of excitations and unit impulse response function of the structure [11,12]. However, these calculations are complex and cumbersome, so that they are seldom used in random analysis for engineering structures.

For this consideration, in most cases, Power Spectrum Density (PSD) is used to characterize the statistical property of random variables, and the system equation is solved in frequency domain by Laplace transform or Fourier transform. The PSD of each structural output can be obtained by multiplying the PSD matrix of excitations and transfer functions of the linear system. To achieve accurate dynamic stresses, the stress transfer functions should be determined by a large number of experiments, thus leading to high expense in research and development. Meanwhile, a large number of matrix multiplications at discrete frequencies also need a significant computational effort.

For a large scale multi-freedom linear system, dynamic responses are usually obtained by means of transformation of coordinates [13–16]. Thus, the system responses are expressed by superposing modal responses. Several computational methods, such as Square-Root-Sum-Squares (SRSS) method, Absolute (ABS) method, and Combined-Quadratic-Combination (CQC) method, are widely used in engineering practice. Using SRSS method, any modal response is assumed to be independent of each other, whilst using ABS method, it is supposed to be precisely correlated to others. These two methods don't take the exact couplings of different modes into account. Comparing with them, CQC method [11,12] accurately considers different modal cross-terms; never-

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theless, the coupling coefficients of cross-terms are complex and time-consuming in calculation. Pseudo excitation method [17–20] provided an effective approach for calculating structural responses under random loading by transferring a stationary random excitation to a harmonic excitation, or transferring a non-stationary random excitation to the deterministic time history.

So far, all the above-mentioned methods are mainly used to calculate structural displacements under random vibration. The dynamic stresses from those methods are usually not accurate, which is crucial for fatigue-life prediction. The reasons may be: 1) usually only a few low modes are included, and the higher modes are neglected in calculation; 2) the stresses are obtained by differentiating the displacement functions.

Aiming at improving the computational efficiency and the accuracy of dynamic stresses induced by random loading, a new method is presented in this paper. After transferring the random loading given by PSD into harmonic loading, the dynamic stresses are obtained by combining the modal stresses superposition with the equivalent treatment for higher modal responses. Thus, this new algorithm can avoid the derivation of displacement functions, and is of high computational efficiency with all the contributions of modal cross-terms and the higher modes which are truncated in usual methods. And also, it is easily programmed for large finite element models of engineering structures.

This paper is organized as follows. Following the introduction, the new computational method is presented in Section 2. In Section 3, as an example, it is applied to predict the dynamic stresses of a thin plate subjected to white noise. Experiments are described in Section 4, with comparison among the predictions using the new method, the experimental results, and the results using CQC method. Section 5 draws the conclusions.

2. New computational method for dynamic stresses under random loading

2.1. Theoretical analysis

Considering a multi-freedom system, its equation of motion under random loading can be expressed as

$$[\mathrm{M}]\{\ddot{y}\}+[\mathrm{C}]\{\dot{y}\}+[\mathrm{K}]\{y\}=\big\{f(t)\big\} \tag{1}$$

where [M], [C], [K] is the matrix of mass, the matrix of damp, and the matrix of stiffness of a structure, respectively; $\{\ddot{y}\}$, $\{\dot{y}\}$, $\{y\}$ is the nodal acceleration, the nodal velocity, and the nodal displacement, respectively. $f(t)$ is the equivalent deterministic excitation obtained from Eq. (2)

$$f(t)=\sqrt{S_f(\omega)}e^{i\omega t} \tag{2}$$

where $S_f(\omega)$ is the PSD of the stationary random loading.

Suppose

$$\{y\}=[\Phi]\big\{q(t)\big\}=\sum_{i=1}^{N}q_i\{\phi\}_i \tag{3}$$

where $q(t)$ is the modal response, $[\Phi]$ is the normalized modal matrix.

Substituting Eq. (3) into Eq. (1), and left-multiplying the equation with $[\Phi]^T$ (T means matrix transpose), it becomes

$$[\bar{\mathrm{M}}]\{\ddot{q}\}+[\bar{\mathrm{C}}]\{\dot{q}\}+[\bar{\mathrm{K}}]\{q\}=[\Phi]^T\big\{f(t)\big\} \tag{4}$$

and

$$[\bar{\mathrm{M}}]=[\Phi]^T[\mathrm{M}][\Phi],$$
$$[\bar{\mathrm{C}}]=[\Phi]^T[\mathrm{C}][\Phi],$$
$$[\bar{\mathrm{K}}]=[\Phi]^T[\mathrm{K}][\Phi].$$

The system equation is decoupled into N independent equations as

$$\ddot{q}_i+2\xi_i\omega_i\dot{q}_i+\omega_i^2q_i=P_i,\quad i=1,2\ldots,N \tag{5}$$

In Eq. (5), ξ_i is the ith modal damping ratio; ω_i is the ith modal frequency; P_i is the ith generalized excitation and $P_i=\{\phi\}_i^T\{f(t)\}$. The steady modal response is obtained from

$$q_i=H_iP_i \tag{6}$$

where H_i is the frequency response function, and $H_i=(\omega_i^2-\omega^2+2i\xi_i\omega_i\omega)^{-1}$.

In common finite element analysis, dynamic stresses are obtained by differentiating displacement functions, which losses more accuracy in the random analysis when the excitation changes drastically and frequently. Therefore, in this study, it is proposed that the dynamic stresses will be determined from the following equation

$$\{\sigma\}=\sum_{i=1}^{N}q_i\big\{\phi^\sigma\big\}_i \tag{7}$$

where $\{\phi^\sigma\}_i$ is the ith modal stresses representing the stress distribution on the structure when only the corresponding modal response is a unit, and other modal responses are zero. This approach can avoid the loss of accuracy induced from differentiating displacement functions, and make it possible to directly analyze dynamic stresses in each concerned position on the structure, which is particularly necessary in stress prediction for vibration fatigue or stress analysis with stress concentration.

Furthermore, the dynamic stresses can be divided into two terms as

$$\{\sigma\}=\sum_{i=1}^{N_d}q_i\big\{\phi^\sigma\big\}_i+\sum_{i=N_d+1}^{N}q_i\big\{\phi^\sigma\big\}_i \tag{8}$$

where the first term is superposition of the low modes, and the second term represents the modal responses of the ignored higher modes from N_d+1 to N. Actually, the contributions of inertia and damping to the higher modal response are smaller comparing with structural stiffness, thus, these higher modal responses could be easily solved by

$$q_i=\frac{P_i}{\omega_i^2},\quad i=N_d+1,\ldots,N \tag{9}$$

Substituting Eq. (9) into Eq. (8), and let $\{\tilde{\sigma}\}_s$ be the global quasi-static response, as

$$\{\sigma\}_s=\sum_{i=1}^{N}\frac{P_i}{\omega_i^2}\big\{\phi^\sigma\big\}_i \tag{10}$$

Then, the dynamic stresses can be determined by

$$\{\sigma\}=\{\sigma\}_s+\sum_{i=1}^{N_d}\big\{\phi^\sigma\big\}_i\left[q_i-\frac{P_i}{\omega_i^2}\right] \tag{11}$$

It is seen that the proposed method given above provides an accurate and efficient approach to obtain the dynamic stresses under random loading. The dynamic stresses are represented by combining a small number of modal stresses with the stresses from the equivalent quasi-static treatment for higher modal responses, so this proposed method can be named as Precise Modal Stresses method. This method eliminates the computational expense on higher modes, but account for their contributions. It is seen from