Aerospace Science and Technology ••• (••••) •••–•••

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Radial basis function mesh deformation based on dynamic control points

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ARTICLE INFO

Article history:
Received 29 October 2016
Accepted 19 January 2017
Available online xxxx

Keywords:
Computational fluid dynamics
Mesh deformation
Radial basis function interpolation
Moving boundaries
Data reduction
Dynamic control point

ABSTRACT

Radial basis function (RBF) interpolation is a robust mesh deformation method, which has the main property of interpolating the displacements of mesh boundary points to the internal points through RBF. However, this method is computationally intensive, especially for problems with large number of grids. To handle this problem, a data reduction RBF method has been developed in literature. By using greedy algorithm, only a small subset of mesh boundary points is selected as the control point set to perform mesh deformation. Subsequently, much few boundary points are needed to approximate the shape of geometry and the computational cost of data reduction RBF method is much lower than the original RBF. Despite the referred benefits, this method incurs the loss of geometry precision especially at boundaries where large deformation happens, which results into the decline of deformation capacity. To further improve the data-reduction RBF method, a novel dynamic-control-point RBF (DCP-RBF) mesh deformation method is proposed in this paper, which employing a dynamic set of control points. In each time step of mesh deformation, the neighboring boundary point near the cell with the worst quality is added into the control point set, while the neighboring control point near the cell with the best quality is removed from the control point set. In this way, it is ensured that there are more control points placed around the region with lower mesh quality, where usually large deformation occurs. Consequently, in contrast with the data reduction RBF method, DCP-RBF permits significantly larger mesh deformation with a quite small increase in computational cost. The superiority of the proposed DCP-RBF method is demonstrated through several test cases including both 2D and 3D dynamic mesh applications.

Introduction

For CFD codes applied to engineering problems, such as aeroelastics, control surface movement and optimization design, motion and deformation of the computational domain frequently occurs. Therefore, it is preferable for adapting the mesh to the changes of the computational domain shape. The literature review shows that mesh deformation methods can be broadly classified into two categories: algebraic methods and partial differential equation (PDE) methods [1]. Algebraic methods include spring analogy method [2–4], transfinite interpolation (TFI) [5], inverse distance weighting interpolation (IDW) [6] and RBF interpolation. The spring analogy method, originally introduced by Batina [3], models each edge of triangle cell of the unstructured mesh by a spring, and the length of the edge is related to the stiffness. Although this method is mathematically simple, mesh-connectivity information is required, and a set of equations must be solved, of which the dimension is equal to the number of all the mesh points. Obviously the method is very expensive for large scale problems. It also tends to produce poor quality mesh when large deformation happens. To overcome this shortcoming, an additional torsional spring is attached to each vertex of the spring system to prohibit the interpenetration of neighboring triangles [4], and consequently lead to larger amount of calculations. The TFI method interpolates the displacements of mesh boundary points to the internal mesh points along the mesh lines. It has high efficiency but is only applicable to structured mesh. The IDW method is a simple and straightforward method [6]. Without solving any equation system, the boundary mesh point displacements are directly interpolated to the interior of the flow domain, and the value of weighting function is directly related to the distance between the mesh interior point and boundary point. Classified into PDE methods, the linear elasticity method is robust, and mesh deformation is treated as analogous to a linear elasticity problem [7,8]. During the process, a linear elasticity equation should be solved. Much like the spring analogy method, the linear elasticity method involves solving an equation

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http://dx.doi.org/10.1016/j.ast.2017.01.022
1270-9638/

system including all the mesh points, so it is also computation-intensive.

RBF interpolation method was first presented by de Boer et al. [9] in 2007. As a point-by-point mesh deformation approach, RBF method requires no mesh-connectivity information and thus is convenient for parallelization. The method is also suitable for both unstructured and structured mesh. Compared with the spring analogy method, RBF method can handle significantly larger deformation and maintain good mesh quality and geometric accuracy, in other words, RBF method is robust. However, for large problems, RBF method is also expensive since a linear equation system needs to be solved, of which the dimension is equal to the number of mesh boundary points. Nowadays, RBF mesh deformation method has been widely used in many fields, such as aircraft control surface deflection [10], flapping wings and rotor hover simulation [11–13], aeroelastics and fluid–structure interaction [14,15], and shape optimization design [16–19]. Some research efforts have been focused on the parallelization of RBF method as well [20,21].

To reduce the cost of RBF method, in the work of Rendall and Allen [22], only a subset of boundary mesh points is selected as the control point set to perform mesh deformation. During the process of mesh deformation, an equation system involving all the control points should be solved, and the computational cost of the system tends to have the magnitude of $O(N^3)$, where N is the number of control points, so in contrast with the pure form RBF method, the data-reduction algorithm is computationally much cheaper. Data-reduction RBF based on the greedy algorithm was investigated and a comparison was made among three different error functions used in the algorithm [23]. Sheng and Allen [1] investigated two greedy algorithms to reduce the number of control points, and then developed a robust, efficient and accurate RBF method. Meanwhile, some recent research efforts focus on easing the stringent requirement of solving the equation system. Gumerov and Duraiswami [24] proposed a fast radial basis function interpolation via the preconditioned Krylov iteration, and the overall computational cost is reduced to $O(N \log N)$. Coulier and Darve [25] proposed an efficient RBF mesh deformation method by means of the inverse fast multipole method and the computational cost scaling is only $O(N)$. There are also some other research work for the improvement of efficiency in the literature, see Ref. [26] and [27].

For the pure data-reduction RBF method, as a result of the reduction of control points, only a small subset of the boundary points is selected as control point set, so the boundary geometry precision declines and the interpolation error is brought in. Then the robustness is weakened and some correction will be needed. T. Gillebaart et al. [28] proposed an "adaptive" RBF mesh deformation method. For the boundary points not included in the control points subset, an explicit boundary correction is applied to obtain the exact boundary representation. A correction function is employed in the correction step to obtain the correctional displacement of each single internal point. Wang et al. [29] proposed a multilevel subspace RBF interpolation method based on a "double-edged" greedy algorithm. This method is computationally efficient and preserves orthogonality.

To further strengthen the robustness of data reduction RBF method, in the present work, a dynamic-control-point RBF method is proposed. The basic procedure can be divided into two steps. First, before mesh deformation, a subset of boundary points is selected as the initial control point set. Then at each time step, after mesh deformation, the neighboring boundary point near the cell with the worst quality is added into the control point set, and the neighboring control point near the cell with the best quality is removed from the control point set. In this way, it is ensured that there are more control points placed around the region with lower mesh quality, where usually large deformation occurs. As a result, significantly larger mesh deformation is allowed with a very small increase in computational cost, comparing with the data-reduction algorithm. Several test cases as representatives of dynamic mesh applications are used to demonstrate the superiority of this method.

1. RBF mesh deformation and data-reduction algorithm

The term RBF refers to a series of functions with its general form as $\Phi(x) = \varphi(\|x\|)$, where $\|x\|$ denotes the Euclidean distance. So the basic variable of RBF is spatial distance. Generally, the distance is normalized through a value r called compact radius: $\xi = \|x\|/r$, and

$$\Phi_r = \varphi(\xi) = \begin{cases} f(\xi) & 0 \le \xi \le 1 \\ 0 & \xi > 1 \end{cases} \tag{1}$$

The influence domain of a control point is inside a circle (for two-dimensional space) or a ball (for three-dimensional space), of which the center is the control point and the radius is the compact radius. The general form of RBF interpolation can be expressed as:

$$F(\boldsymbol{r}) = \sum_{j=1}^{n_b} \alpha_j \varphi\left(\|\boldsymbol{r} - \boldsymbol{r}_{b_j}\|\right) + p(\boldsymbol{r}) \tag{2}$$

where $F(\boldsymbol{r})$ is the interpolation function, n_b is the number of control points used in the interpolation, and α_j is the weighting coefficient of the jth control point. The general form of RBF is $\varphi(\|\boldsymbol{r} - \boldsymbol{r}_{b_j}\|)$, where $\|\boldsymbol{r} - \boldsymbol{r}_{b_j}\|$ is the distance between position vector $\boldsymbol{r}$ of the internal mesh point and position vector $\boldsymbol{r}_{b_j}$ of the jth control point. The $p(\boldsymbol{r})$ is a first-order polynomial. There are additional requirements

$$\sum_{j=1}^{n_b} \alpha_j q(\boldsymbol{r}_{b_j}) = 0 \tag{3}$$

for all polynomial q with a degree no more than that of p. As the result of using a linear polynomial, rigid body translations are exactly recovered [9]. There are many types of RBFs, for example, Wendland's C2 [30] is usually selected to perform mesh deformation:

$$f(\xi) = (1-\xi)^4(4\xi + 1) \tag{4}$$

With the displacements of the control points (namely the boundary mesh points), the displacements of internal mesh points can be interpolated. The interpolation weighting coefficient α_j and the coefficients of each item of polynomial $p(\boldsymbol{r})$ depend on the consistency of interpolation of mesh boundary points. In other words, the interpolated function values at all the control points are recovered exactly, which are equal to the boundary mesh displacements. So Eq. (5) must be satisfied:

$$\begin{bmatrix} \mathbf{M} & \mathbf{P}_b \\ \mathbf{P}_b^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{d}_b \\ \mathbf{0} \end{bmatrix} \tag{5}$$

where the square matrix $\mathbf{M}$ can be expressed as:

$$\mathbf{M} = \begin{bmatrix} \varphi_{b_1 b_1} & \varphi_{b_1 b_2} & \cdots & \varphi_{b_1 b_{n_b}} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{b_{n_b} b_1} & \varphi_{b_{n_b} b_2} & \cdots & \varphi_{b_{n_b} b_{n_b}} \end{bmatrix} \tag{6}$$

and there is $\varphi_{b_i b_j} = \varphi(\|\boldsymbol{r}_{b_i} - \boldsymbol{r}_{b_j}\|)$, namely the RBF based on the distance between the ith and jth control point.

$\mathbf{P}_b$ is a $n_b \times 4$ matrix containing the coordinates of the control points: