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Online optimal midcourse trajectory modification algorithm for hypersonic vehicle interceptions

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A R T I C L E I N F O

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A B S T R A C T

This paper presents an online optimal midcourse trajectory modification algorithm for the hypersonic interceptions based on the neighboring optimal control theory, which takes the current states deviations and the revised terminal constraints as inputs and generates the optimal control modifications. Firstly, the midcourse guidance for hypersonic interception is introduced as an optimal control problem, which is solved with the well-developed Gauss Pseudospectral Method (GPM) to generate a nominal trajectory that satisfies the first order optimality conditions. Secondly, the Neighboring Optimal Trajectory Existence Theory (NOTET) is given and the first order optimality equations are further differentiated to second order to acquire the control modifications. The perturbations of the terminal co-states are expressed with the current states perturbations and the revised terminal constraints by solving the second order equations and inversely integrating the perturbation dynamic equations. The optimality of the proposed algorithm is proved. Finally, five different interception scenarios are simulated and comparisons are made with particle swarm optimization (PSO) method and GPM. Simulation results reveal that the proposed method has the merits of high precision equivalent with GPM and better computing efficiency than PSO and GPM, which testifies the effectiveness and online application feasibility.

1. Introduction

The hypersonic vehicles in near space, which are traditionally characterized with high velocity over Mach 5, potential longitudinal and cross maneuverable footprint over thousand miles, have the advantages of stealthy and penetration abilities over the existing flight vehicles [1,2]. More and more countries are developing such vehicles as carriers or weapons to gain dominance in the near space. However, a successful interception against such targets is a challenging task. In order to achieve the direct collision performance, the interceptors with more advanced and precise guidance and control systems are in crucial demand. The high velocity of the targets compels the interceptors to acquire a relevant or even a superior velocity, which naturally results in the physical limitations such as the dynamic pressure, overload, and control input saturations. The traditional midcourse guidance law such as the proportional navigation guidance and its modified versions can hardly meet these demands. What's more, the unpredictable trajectory caused by the potential maneuverability of the targets results in a rapidly modified predicted impact point (PIP) rather than a relatively fixed one as in the conventional interception. The interceptors should be able to fly towards the always changing PIP while satisfying the physical constraints at the same time, which should be treated as an online trajectory modification or trajectory regeneration problem rather than the conventional trajectory following one.

However, the accomplishment that takes all of the challenges into consideration is relatively little, and most of the current studies just focused on one of the aspects. For example, many experts have carried out the research about the real-time trajectory generation problem. The representative accomplishments can be found by Faiz and Lwis with the differential flatness method [3,4], Yang with the polynomial parameterization method [5], Verma with the inverse dynamics method [6], Doebbler with the discrete-node heuristic search method [7], Dever with the trajectory interpolation method [8], Henshaw with the indirect collocation method [9], Somavarapu with the direct collocation method [10], Wang and Zhao with the particle swarm optimization method [11,12]. The recently well-developed Gauss Pseudospectral Method (GPM) has also found its way in the online trajectory optimization area [13–15] with its fast convergence and optimality characteristic [16]. However, the researches aforementioned have testified the online trajectory generation, but failed to consider the trajectory modification or regeneration problem. In this aspect, some experts

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attempt to introduce the receding horizon control method [17,18], and the model predictive static programming method [19–21] to iteratively detect the changes of the terminal trajectory destinations, and some other experts use the polynomial function [22], multiple shooting method [23] and particle swarm optimization method [24] for the trajectory reshaping. These accomplishments achieve the trajectory regeneration at the cost of totally deserting the former nominal one and thus put a crucial burden on the computing efficiency and onboard computation. In fact, the former nominal trajectory could be used to generate a new trajectory with the neighboring optimal control theory if the revised terminal constraints or the perturbed states are not far from the nominal ones. Yan [25,26] and Tian [27] have designed a robust state feedback guidance law with the indirect Legendre pseudospectral feedback method to negate the initial trajectory perturbations. However, in case of the terminal states modifications, their accomplishments fail to achieve satisfactory results. Xu and Huang [28–31] have further studied the hypersonic flight control systems under circumstances of actuator fault and dead-zone input nonlinearity with back-stepping neural network design, which give insight into the online control command realization for hypersonic interceptors.

In terms of the midcourse trajectory modification demanded by the advanced interceptors, an online optimal midcourse trajectory modification algorithm is designed in this paper that makes full use of the former nominal trajectory information. A further leap is made compared with Refs. [25–27] by taking both of the current states perturbations and the revised terminal constraints as feedback inputs to allow for the terminal position modifications. The control modifications are then generated by further differentiating the first order optimality conditions instead of searching for a proper control at a vast allowable domain as done in the conventional researches [17–24]. The difficulty of expressing the perturbations of the co-states is resolved by solving the terminal second order equations and inversely integrating the perturbation dynamic equations to the initial moments. Simulation results show that the algorithm has the merits of high precision equivalent with the GPM and better computing efficiency than GPM and PSO.

The remainder of the paper is organized as follows. The midcourse trajectory generation mathematical model is introduced as an optimal control problem in Section 2. In Section 3, the GPM is introduced for the generation of the offline nominal trajectory that satisfies the first order optimality conditions. Section 4 describes the online trajectory modification algorithm in detail with the neighboring optimal control theory and in Section 5 the simulation results are presented to prove the effectiveness of the proposed method. Section 6 gives the conclusions.

2. Problem formulation

2.1. Interceptor dynamic model

The interceptor considered in this research is assumed to be a non-rotational rigid body, of which the dynamic equations in the planner and horizontal plane are similar and thus can be separated. Without loss of generality, the planner point-mass kinematic equations are considered, which can be described as follows:

$$\dot{V}=\frac{P\cos\alpha-C_xqS}{m}-g\sin\theta \tag{1}$$

$$\dot{\theta}=\frac{P\sin\alpha+C_yqS}{mV}-\frac{g\cos\theta}{V} \tag{2}$$

$$\dot{x}=V\cos\theta \tag{3}$$

$$\dot{y}=V\sin\theta \tag{4}$$

where V is the velocity of the interceptor. P is the thrust value, g is the gravitational acceleration. m is the total mass of the interceptor. θ is the flight path angle. q is the dynamic pressure. S is the reference area. x and y are the locations in the inertial coordinate. C_x and C_y are the drag coefficient parameter and the lift coefficient parameter respectively, which have the mathematical form of

$$C_y=C_y^{\alpha}(Ma)\alpha \tag{5}$$

$$C_x=C_{x0}(Ma)+K(Ma)C_y^2 \tag{6}$$

where α is the angle of attack, Ma is the Mach number. C_y^{α} is the partial derivative of C_y in respect to α. C_{x0} and K are zero-lift drag coefficient and induced drag coefficient. Noticing that C_y^{α}, C_{x0} and K are all functions of the Mach number.

In order to ensure the trajectory's smoothness and the online realization, several constraints should be considered in the trajectory generation process, such as the dynamic pressure constraints:

$$q=\frac{1}{2}\rho V^2\le q_{\max} \tag{7}$$

where $q_{\max}$ is the maximum dynamic pressure, ρ is the air density as a function of the altitude expressed with Eq. (8):

$$\rho=\rho_0 e^{-\frac{y}{y_r}} \tag{8}$$

where $\rho_0=1.2250\ \mathrm{kg/m^3}$, $y_r=7254.3$ m. The control input α should be constrained within the physical bound:

$$\|\alpha\|\le\alpha_{\max} \tag{9}$$

The constraint of the overload n is denoted as:

$$n=\frac{\sqrt{(C_xqS)^2+(C_yqS)^2}}{mg}\le n_{\max} \tag{10}$$

2.2. Trajectory optimization problem

As the most time consuming phase in the interception process, the midcourse guidance is of great importance to deliver the interceptor to a specified area for further midcourse and terminal guidance handover. The final states of the midcourse guidance serve as the initial states of the terminal guidance. As to the hypersonic vehicle interception scenarios, the adjusting time left for the terminal guidance is very short and if the initial states diverge far from the desired ones, the kinematic kill performance can hardly be achieved. So the control objective of the midcourse guidance is selected as the PIP with specified terminal flight path angle constraint and the maximum terminal velocity is preferred. The criteria function J can be formulated as:

$$J=\phi\big(\mathbf{X}(t_f),t_f\big)=-V_f \tag{11}$$

where $\mathbf{X}=[V\quad\theta\quad x\quad y]^{\mathrm{T}}$. The terminal cost function is described as:

$$\boldsymbol{\psi}=[\theta-\theta_f\quad x-x_f\quad y-y_f]^{\mathrm{T}}=\mathbf{0} \tag{12}$$

The path constraints with Eqs. (7)–(10) can be stated as:

$$\mathbf{C}\big(\mathbf{X}(t),u(t),t\big)\le\mathbf{0} \tag{13}$$

By introducing the Lagrange parameters $\boldsymbol{\nu}=[\boldsymbol{\nu}_{\theta}\quad\boldsymbol{\nu}_x\quad\boldsymbol{\nu}_y]^{\mathrm{T}}$, the auxiliary function $\boldsymbol{\Phi}$ can be written as:

$$\boldsymbol{\Phi}=\phi+\nu^{\mathbf{T}}\boldsymbol{\psi} \tag{14}$$

The Hamilton function of the trajectory optimization problem $\mathbf{H}$ can be formed as:

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