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A review of uncertainty propagation in orbital mechanics

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ABSTRACT

Orbital uncertainty propagation plays an important role in space situational awareness related missions such as tracking and data association, conjunction assessment, sensor resource management and anomaly detection. Linear models and Monte Carlo simulation were primarily used to propagate uncertainties. However, due to the nonlinear nature of orbital dynamics, problems such as low precision and intensive computation have greatly hampered the application of these methods. Aiming at solving these problems, many nonlinear uncertainty propagators have been proposed in the past two decades. To motivate this research area and facilitate the development of orbital uncertainty propagation, this paper summarizes the existing linear and nonlinear uncertainty propagators and their associated applications in the field of orbital mechanics. Frameworks of methods for orbital uncertainty propagation, the advantages and drawbacks of different methods, as well as potential directions for future efforts are also discussed.

1. Introduction

The term space situational awareness (SSA) refers to the ability to view, understand and predict the physical location of natural and manmade objects in orbit around the Earth, with the objective of avoiding collisions [1]. In recent years, SSA has gained increasing attentions as the number of the tracked resident space objects (RSOs) continues to grow rapidly. By December 6, 2016, the number of space objects with sizes larger than 10 cm is beyond 17,000, with 76% of them being space debris (Table 1). SSA is the comprehensive knowledge of the near-Earth space environment through the tracking and identification of orbiting space objects in order to protect space assets and maintain awareness of potentially adversarial space deployments. The proper characterization of uncertainty in the orbital state of a space object is essential to many SSA functions including tracking and data association, conjunction analysis and probability of collision, sensor resource management, and anomaly detection [2].

The efficient and accurate propagation of uncertainty in orbit for long time durations is an important issue in SSA. The main reason is that the number of RSOs of interest, typically defined to be equal to or larger than the size of a softball (~10 cm), is significantly greater than the number of sensors available for tracking them. Only sparse measurements can be expected for a given object. Thus, it is often required to propagate uncertainty for multiple orbits, possibly spanning several days without measurement.

In the field of space trajectory design and operation, uncertainty propagation usually refers to the orbital uncertainty propagation, that is to determine the satellite's state moments (generally mean and covariance matrix) or probability density function (PDF). In the last forty years of the 20th century, orbital uncertainty propagations are usually addressed by linear models or nonlinear Monte Carlo (MC) simulations [3]. The linear methods are highly efficient as they linearize the problem, but their accuracy declines in highly nonlinear systems or long-duration propagations. On the other hand, the MC simulation provides high-precision results, but is computationally expensive. To avoid these shortcomings, many analytical or semi-analytical techniques for nonlinear orbital uncertainty propagation have been developed in recent years.

This review provides a clearly categorized survey for the existing uncertainty propagation methods and their associated applications. Some related problems which have not been well addressed are also discussed. This paper is by no means exclusive, and we apologize to authors and readers for whose work that have not been included. The outline of the paper is structured as follows. Section 2 presents some theory preliminaries related to uncertainty and makes an introduction for the orbital uncertainty propagation problem. Section 3 gives an overall review and category for different uncertainty propagation methods. Applications of uncertainty propagation in SSA and space flight mission designs are discussed in Section 4. Furthermore, Section 5 discusses unsolved problems and future research directions for orbital uncertainty quantification. Finally, conclusion is drawn in Section 6.

2. Theory preliminaries

For convenience of discussion, we begin with the basic conceptions

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Table 1
Orbital population till December 6, 2016[a].

Status	Payloads	Debris	Total
On Orbit	4,269	13,587	17,856
Decayed	3,194	20,826	24,020
Total	7,463	34,413	41,876

[a] URL: http://www.celestrak.com/satcat/boxscore.asp [retrieved 6 December 2016].

and definitions relevant to uncertainty propagation in this section.

2.1. Probability theory and stochastic process

2.1.1. Stochastic dynamics equation

Orbital dynamic problem entailing uncertainty can be expressed by the Itô stochastic differential equation [4],

$$d\boldsymbol{x}(t) = \boldsymbol{f}(\boldsymbol{x}, t)dt + \boldsymbol{G}(t)d\boldsymbol{\beta}(t) \quad (1)$$

where $\boldsymbol{x}\in\mathbb{R}^n$ is the random state vector, $\boldsymbol{\beta}(t)\in\mathbb{R}^m$ is a m-dimensional Brownian motion process with zero mean and covariance $\boldsymbol{Q}(t)$. The vector function $\boldsymbol{f}(\boldsymbol{x}, t)$ captures the deterministic part of the dynamics, and $\boldsymbol{G}(t)$ is an $n\times m$ matrix characterizing the diffusion. The initial condition uncertainty is determined by the PDF $p(\boldsymbol{x}_0, t_0)$, assumed known.

In this study, the deterministic part $\boldsymbol{f}(\boldsymbol{x}, t)$ of the dynamics concerns the satellite's motion under a non-spherical gravitational force field, which is given by an ordinary differential equation (ODE) [5]

$$\boldsymbol{f}(\boldsymbol{x}, t) = \ddot{\boldsymbol{r}} = -\mu\frac{\boldsymbol{r}}{r^3} + \boldsymbol{a}_{per} + \boldsymbol{\Gamma} \quad (2)$$

where $\boldsymbol{x}\in\mathbf{R}^n$ denotes the satellite's state with dimensionality of n=6, $\boldsymbol{x} = [\boldsymbol{r}, \boldsymbol{v}]^T$, $\boldsymbol{r}$ and $\boldsymbol{v}$ are the position and velocity vectors described in the central body's inertial Cartesian frame, respectively; μ is the central body's gravitational constant, $r=||\boldsymbol{r}||$ and $||\cdot||$ denotes the Euclid norm of a vector; $\boldsymbol{a}_{per}$ is the perturbation acceleration caused by factors such as non-spherical gravity and the atmospheric drag, and $\boldsymbol{\Gamma}$ is the thrust acceleration vector. If the impulsive maneuver is assumed, the thrust acceleration can be approximated in terms of m impulses as follows:

$$\boldsymbol{\Gamma}(t) = \sum_{l=1}^{m}\Delta\boldsymbol{v}_l\delta(t - t_l) \quad (3)$$

where m is the number of impulses, t_l is the l^{th} maneuver time, and $\delta(t - t_l)$ is the delta function.

For a given initial condition $\boldsymbol{x}_0 = \boldsymbol{x}(t_0)$, the solution to Eq. (2) can be implicitly denoted as

$$\boldsymbol{x}(t) = \boldsymbol{\phi}(t; \boldsymbol{x}_0, t_0) \quad (4)$$

2.1.2. Fokker–Planck equation

For a given n-dimensional continuous random vector $\boldsymbol{x}\in\mathbb{R}^n$, the probability of $\boldsymbol{x}$ in some volume Ξ can be defined as:

$$\Pr(\boldsymbol{x} \in \Xi) = \int_{\Xi} p(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} \quad (5)$$

where $p(\boldsymbol{x}, t)$ is the PDF of random vector $\boldsymbol{x}$. It should be noticed that the PDF must be nonnegative for all $\boldsymbol{x}$ and it also must integrate to unity over the state space.

For a given dynamical system that satisfies the Itô stochastic differential equation, the time evolution of a PDF $p(\boldsymbol{x}, t)$ over $\boldsymbol{x}$ at time t is described by the Fokker–Planck equation (FPE) [6]:

$$\frac{\partial p(\boldsymbol{x}, t)}{\partial t} = -\sum_{i=1}^{n}\frac{\partial}{\partial x^i}[p(\boldsymbol{x}, t)\boldsymbol{f}^i(\boldsymbol{x}, t)] + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2}{\partial x^i\partial x^j}\{p(\boldsymbol{x}, t)[\boldsymbol{G}(t)\boldsymbol{Q}(t)\boldsymbol{G}^T(t)]^{ij}\} \quad (6)$$

The FPE is a partial differential equation that satisfies the propagation of a PDF. Hence, the solution of the FPE provides a complete statistical description of a trajectory governed by Eq. (1). However, solving the FPE in orbital mechanics is a difficult task, primarily because the underlying FPE is defined in a relatively high dimensional state–space (6-D) and is driven by the nonlinear perturbed Keplerian dynamics, as shown in Eq. (2). Therefore, most of the previous uncertainty quantification algorithms make extensive use of Gaussian and local linearity assumptions, so that only lower-order statistical moments require propagating, especially the mean vector and covariance matrix.

2.1.3. Statistical moments

In statistics, the moment of PDF is a specific quantitative measure of the shape of a set of points. Particularly, the moments up to fourth-order, i.e., mean, variance, skewness, and kurtosis, provide information related to the shape of a PDF. The mean is used to refer to one measure of the central tendency of a probability distribution, the variance measures how far a set of random numbers are spread out from their mean, the skewness is a measure of symmetry (or more precisely, the lack of symmetry), and the kurtosis is a measure of whether the data are heavy-tailed or light-tailed relative to a normal distribution.

Given a random vector $\boldsymbol{x}\in\mathbb{R}^n$ with a PDF $p(\boldsymbol{x}, t)$, the mean and covariance matrix are defined as [4]:

$$\begin{aligned}\boldsymbol{m}(t) &= E[\boldsymbol{x}(t)] = \int_{\infty}\boldsymbol{\xi}p(\boldsymbol{\xi}, t)d\boldsymbol{\xi} \\ \boldsymbol{P}(t) &= E[(\boldsymbol{x} - \boldsymbol{m})(\boldsymbol{x} - \boldsymbol{m})^T] = \int_{\infty}(\boldsymbol{\xi} - \boldsymbol{m})(\boldsymbol{\xi} - \boldsymbol{m})^T p(\boldsymbol{\xi}, t)d\boldsymbol{\xi}\end{aligned} \quad (7)$$

where $E[\cdot]$ represents the expectation operator. Note that the variances appear along the diagonal elements of the covariance matrix, i.e., $\boldsymbol{\sigma}^2 = \mathrm{diag}(\boldsymbol{P})$; $\boldsymbol{\sigma}$ is the standard deviation and diag(·) denotes to extract the diagonal of a matrix or to build a matrix with assigned diagonal.

Differentiating the mean and covariance matrix in Eq. (7), we get:

$$\begin{aligned}\dot{\boldsymbol{m}}(t) &= E[\dot{\boldsymbol{x}}(t)] = E[\boldsymbol{f}(\boldsymbol{x}, t)] \\ \dot{\boldsymbol{P}}(t) &= E[\dot{\boldsymbol{x}}\boldsymbol{x}^T + \boldsymbol{x}\dot{\boldsymbol{x}}^T] - (\dot{\boldsymbol{m}}\boldsymbol{m}^T - \boldsymbol{m}\dot{\boldsymbol{m}}^T)\end{aligned} \quad (8)$$

Similarly, the third-order standardized moment, skewness $\boldsymbol{S}^i$, and the fourth-order standardized moment, kurtosis $\boldsymbol{K}^i$, can be defined as [4]:

$$\begin{aligned}\boldsymbol{S}^i &= \frac{E[(\boldsymbol{x}^i - \boldsymbol{m}^i)^3]}{(\boldsymbol{\sigma}^i)^3} \\ \boldsymbol{K}^i &= \frac{E[(\boldsymbol{x}^i - \boldsymbol{m}^i)^4]}{(\boldsymbol{\sigma}^i)^4} - 3\end{aligned} \quad (9)$$

2.1.4. Gaussian probability density function

In practice, we usually assume the unknown uncertainties as Gaussian distribution. The Gaussian PDF for an n-dimensional Gaussian random vector $\boldsymbol{x}\sim p_g(\boldsymbol{x}; \boldsymbol{m}, \boldsymbol{P})$ is defined as:

$$p_g(\boldsymbol{x}; \boldsymbol{m}, \boldsymbol{P}) = \frac{1}{\sqrt{(2\pi)^n\det(\boldsymbol{P})}}\exp\{-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{m})^T\boldsymbol{P}^{-1}(\boldsymbol{x} - \boldsymbol{m})\} \quad (10)$$

where det(·) denotes the determinant of a square matrix and exp(·) denotes the exponential function.

An important property of the Gaussian distribution is that the statistics of the Gaussian random vector $\boldsymbol{x}$ can be completely described by the first two moments, i.e., $\boldsymbol{m}$ and $\boldsymbol{P}$. The higher moments of $\boldsymbol{x}$, such as $E[\boldsymbol{x}^i\boldsymbol{x}^j\boldsymbol{x}^k]$ and $E[\boldsymbol{x}^i\boldsymbol{x}^j\boldsymbol{x}^k\boldsymbol{x}^l]$, can be computed as functions of $\boldsymbol{m}$ and $\boldsymbol{P}$, e.g., the first four moments of the Gaussian probability density