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Efficient processing of water wave records via compressive sensing and joint time-frequency analysis via harmonic wavelets

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ABSTRACT

A methodology is proposed for efficient processing of sea wave field data via compressive sensing (CS), and joint time-frequency analysis via harmonic wavelets (HWs) based evolutionary power spectrum (EPS) estimation. In this regard, it is possible to record and store relatively long wave data sequences, whereas the commonly adopted in-practice assumption of stationary data is abandoned. Currently, most wave records are measured by buoys, which acquire data for a time interval representative of stationary time series. Next, following a Fourier transform processing, only few spectral parameters are stored. Thus, detailed information about localized-in-time phenomena are completely lost. Herein, it is shown that CS can be adopted for efficiently compressing and reconstructing wave data, while retaining localized information. For this purpose, CS is used in conjunction with a HW basis for processing long time series. In particular, storage capacity demands are drastically decreased as only the HW coefficients need to be saved. These are determined from a randomly-sampled record by invoking a $L_{1/2}$ norm minimization procedure. The resulting reconstructed record, being longer than conventional wave time series, can no longer be regarded as stationary; thus, a HW based EPS estimate is employed for describing the joint time-frequency features of the record. Finally, the reliability of the methodology is assessed by analyzing wave field data measured at the Natural Ocean Engineering Laboratory (NOEL) of Reggio Calabria. Specifically, comparisons between original and reconstructed records demonstrate a satisfactory agreement regarding the time-histories, and the estimated EPS and relevant statistical quantities, even for up to 60% missing/removed data.

1. Introduction

The concept of sea state has been instrumental in marine engineering, as it has facilitated the development of most modern analysis techniques within the context, for instance, of wave statistics, structural reliability, and mechanics of extreme waves. The primary assumption behind this concept relates to the fact that the free surface displacement recorded during a certain time interval (of order of 100–200 waves) can be construed as part of a realization of a stochastic process with given probability density function (pdf) and power spectral density (psd) [1]. In this context, although Gaussian processes have certainly been the most utilized for determining sea wave statistics, non-Gaussian models are also quite established for describing non-linear phenomena [2,3].

Nevertheless, the underlying assumption of stationarity has never really been questioned by the majority of researchers. In this regard, some observations were noted by Liu et al. [4,5]. Specifically, they observed that conventional approaches used for describing the wave growing process are limited in the sense that they are unable to describe time-localized mechanisms such as wave grouping or wave breaking. This inadequacy of the approaches also relates to the fact that conventional observation intervals are limited to 20–30 min windows, a widely utilized sea state duration [6]. Extending the recording time window is certainly attractive because of the possibility of acquiring more information about the recorded physical processes. However, this extension must cope with additional challenges. First, new tools are necessary for processing longer data sequences due to the increased storage capacity demands, and second, the measured extended signal may exhibit time-varying statistics, and thus, cannot be construed as a realization compatible with an underlying stationary stochastic process.

To address the above challenges, this paper employs and assesses the capabilities of two potent tools/concepts for treating

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long, non-stationary, free surface data sequences; that is, compressive sensing (CS), and evolutionary power spectrum (EPS).

Compressive sensing circumvents the limitation posed by the Nyquist-Shannon sampling rate theorem [7,8] and can be used for acquiring/sampling and storing longer free surface time histories. Currently, wave data are recorded mainly by buoys equipped with accelerometers, which measure the acceleration and provide the free surface displacement by numerical integration [9]. Typically, data are recorded with a sampling interval that varies between 0.5 and 1 s (depending on wave characteristics at the considered site) for a duration of 20–30 min, whereas the time interval between successive records may vary from half an hour to three hours. Next, the Fourier transform is applied on each record of sea surface elevation, and only spectral data including significant wave height H_s and peak spectral period T_p are provided to the final users. The choice of the record duration relates to the definition of sea state. In this regard, it is long enough to be representative of the sea state conditions, but short enough at the same time to guarantee stationarity of the process. Further, the time interval between successive records is chosen to be short enough to capture adequately the long-term variability of the sea conditions. Obviously, this procedure is theoretically consistent with the sea state concept, but it is also dictated by the need for limiting the amount of stored data, and ultimately, lowering the equipment cost.

However, if only synthetic parameters are stored, information about local phenomena, such as freak waves, is completely lost [10]: for this reason, storing full data records is desirable. In this context, considering that sea waves are characterized by a relatively small number of dominant frequencies when expanded in the frequency domain, the CS approach [11,12] can be applied to reconstructing signals that contain sampling gaps (either deliberately for storage capacity purposes or due to equipment failure) in the time domain by selecting an appropriate basis [13–16].

Further, EPS were introduced for describing the time-varying frequency content of non-stationary stochastic processes [17]. The problem of treating non-stationary signals was initially addressed by the short time Fourier transform [18–20] and, later, by the wavelet transform (WT) [21,22] among other alternatives. In this regard, Newland [23–25] introduced the family of generalized harmonic wavelets (GHWs), which was employed in Spanos et al. [26] to estimate the EPS of non-stationary stochastic processes from available realizations. Thus, a GHW-based EPS estimation technique was developed. Further, GHWs have proven to be efficacious for structural dynamics-related applications due to their non-overlapping, box-shaped frequency spectra, their orthogonality properties, and the convenience of combining harmonic balance with statistical linearization techniques [27,28]. Applications of wavelets to ocean engineering related problems are increasing considerably, due to their versatility. For instance, the work of Massel [29] demonstrated the capability of the wavelet transform to provide a time-frequency representation of wave signals. Other contributions relate to studies on wave breaking [30], occurrence of abnormal waves [31] and time series forecasting [32,33].

In the ensuing sections, it is demonstrated that a) GHWs can be used in conjunction with the concept of EPS for processing and capturing time-varying features of free surface elevation records, and b) CS can be combined with a GHW basis for compressing free surface records. Finally, the theoretical developments are exemplified by analyzing long experimental free surface displacement data.

2. Processing and analysis of non-stationary time series

In this section the key elements related to the implementation of the EPS estimation and the CS based reconstruction are delineated. These two tools are strictly connected to each other in the ensuing implementation via the representation adopted for the time series analyses. Indeed, the EPS estimation is pursued via the GHWT, which is invoked also during the signal reconstruction. Other representations can be utilized, as well. However, this one is adopted because it ensures non-overlapping intervals at different scales along the frequency axis, and thus, desirable orthogonality properties hold true.

2.1. Evolutionary power spectrum estimation via the GHWT

The wavelet transform of a finite energy stochastic process $f(t)$ provides a time-frequency representation of $f(t)$. Its calculation relates to the determination of a series of wavelet coefficients at different scales j and time positions k. Note that the scale parameter is related to the frequency, whereas the general form of a continuous wavelet transform of a stochastic process $f(t)$ is given by

$$[W_\Psi f](j,k) = \frac{1}{|j|^{1/2}} \int_{-\infty}^{+\infty} f(t)\Psi^*\left(\frac{t-k}{j}\right)\mathrm{d}t. \tag{1}$$

In Eq. (1) $[W_\Psi f](j,k)$ is the wavelet coefficient at scale j and time position k, the function $\psi(t)$ is the mother wavelet and the symbol (*) denotes the complex conjugate of $\psi(t)$. Eq. (1) represents a convolution operation between $f(t)$ and the basis functions obtained by properly scaling and translating the mother wavelet. The wavelet coefficients provide a measure of the similarity between $f(t)$ and the wavelet. Thus, the higher the correlation is, the larger the coefficient will be.

Focusing on the specific family of harmonic wavelets, they are defined to have a band limited spectrum, whereas two indices (m,n) are used to define the frequency bands and to control their frequency content. Herein, the generalized harmonic wavelet is considered, which is expressed in the frequency domain as [24]:

$$\Psi_{(m,n),k}(\omega) = \begin{cases} \dfrac{1}{(n-m)\Delta\omega} e^{-\dfrac{i\omega k T_0}{n-m}}; & \text{for } m\Delta\omega \le \omega < n\Delta\omega \\ 0; & \text{elsewhere} \end{cases} \tag{2}$$

where T_0 is the total time duration of the signal under consideration, m and n are integer numbers defining the frequency band ($n > m$), and $\Delta\omega = 2\pi/T_0$. The complex harmonic wavelet coefficients are given by:

$$[W^G_{(m,n),k} f(t)] = \frac{(n-m)}{T_0} \int_{-\infty}^{+\infty} f(t)\Psi^*_{(m,n),k}(t)\,\mathrm{d}t, \tag{3}$$

and the evolutionary power spectrum can be estimated as [26]

$$S_f(\omega,t) = S^f_{(m,n),k} = \frac{E(|[W^G_{(m,n),k} f(t)]^2|)}{(n-m)\Delta\omega}; \quad \text{with} \begin{cases} m\Delta\omega \le \omega < n\Delta\omega, \\ \dfrac{kT_0}{(n-m)} \le t < \dfrac{(k+1)T_0}{(n-m)}. \end{cases} \tag{4}$$

The estimation of the EPS of a non-stationary stochastic process $f(t)$ via Eq. (4) requires the calculation of the wavelet coefficient by means of Eq. (3) based on an available ensemble of process realizations. From a practical point of view it is worth noting that the GHWT can be numerically determined by utilizing the Fast Fourier Transform (FFT), which offers significant computational advantages [34].