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# On the formulation of a finite element method for the stiffened multi-layered airfoil/hydrofoil structure: Post buckling analysis for the wings of underwater gliders

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ABSTRACT

A finite element method is developed for the stiffened multi-layered airfoil/hydrofoil structure for the large deformation and finite strain problem. The kinematics of the airfoil/hydrofoil is set up. The Consistent Orthogonal Basis Function Space is applied for the airfoil/hydrofoil structure. Given the airfoil/hydrofoil configuration and boundary conditions, the basis function space can be uniquely determined, such that the diagonal mass matrix is obtained accurately and the basis functions are very identical with the mode shape functions of the structure. In order to satisfy the displacement compatibility condition between adjacent layers of the airfoil/hydrofoil, the traction degree-of-freedom is also induced.

The post buckling analysis is presented for the wing (hydrofoil) structure of the underwater glider. The water pressure is applied on the outer surface and the critical buckling pressure is calculated. The post buckling equilibrium path is also given. The results are verified with ANSYS. The present study of the buckling analysis of the airfoil/hydrofoil under water pressure is helpful to the design of underwater glider.

## 1. Introduction

Underwater glider vehicle has been applied in the deep ocean environment in many applications. The airfoil/hydrofoil structure, as the wing of the underwater glider, plays an important role for the design of the vehicle. In the deep ocean environment, the water pressure is very high. Thus, the buckling analysis of the airfoil/hydrofoil under water pressure is focused here.

In this paper, a high-order dedicated finite element method is developed for the stiffened multi-layered airfoil/hydrofoil structure with smooth configuration. The advantages of the method are:

(1) The exact smooth configuration is obtained;
(2) The order of the basis functions for displacement is high;
(3) The number of DOF is much fewer than that of traditional finite element method;

The research of underwater gliders and finite element applications are reviewed here. The underwater glider patent is first proposed in 1964 [1]. The underwater glide has wide applications in Ocean Engineering. The ocean engineering glider technology background is referred to [2]. In [3], the concept design of the underwater vehicle is presented. The finite element method is suitable to analyze the airfoil/hydrofoil structure. In [4], a beam finite element model is developed for the blade structure. The ABAQUS finite element code is used to study an airfoil-like structure in [5]. The finite element method is used to optimize the airfoil-like structure in [6]. In [7], the finite element method that is focused on coupling of torsion and bending is presented. For the background of nonlinear finite element, it is referred to the book [8]. The hydrodynamics character of the underwater glider is studied in [9]. A rigid body dynamics and control model is presented in [10] for underwater gliders. In [11], the pipe element is developed for the pipe structure with arbitrary variable cross-section. In [12], the pipe element is generalized to dynamical analysis, where the mass matrix is diagonal. In [13], the pipe element

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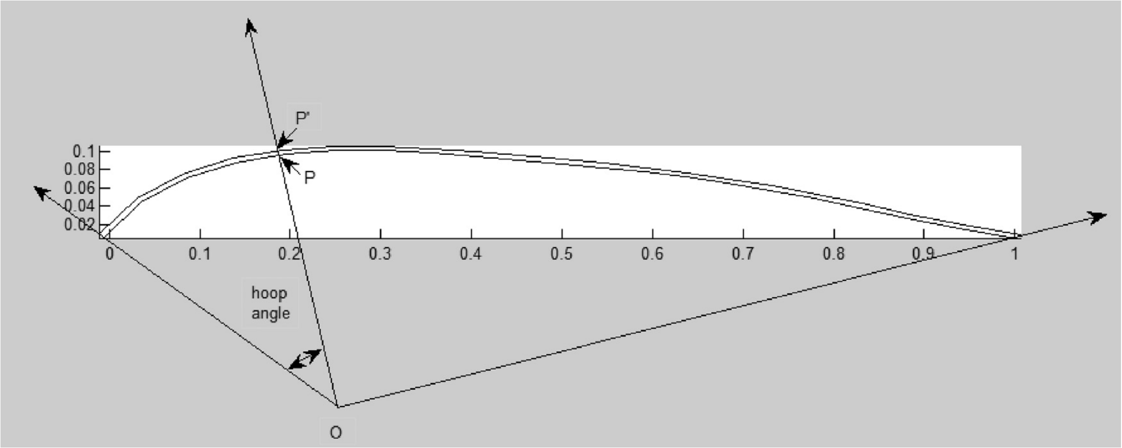

**Fig. 1.** The cross-section of the airfoil/hydrofoil structure.

is generalized to curved pipes. The pipeline can be defined as any 3-D curves and the cross-section is still variable and arbitrary. In [14], the pipe element is generalized to multi-layered pipe case, where the composite pipe is analyzed.

In this paper, the post buckling analysis for hydrofoil structure is presented by using the proposed finite element method. We developed several models, including:

(1) Single-layered airfoil/hydrofoil;
(2) Multi-layered airfoil/hydrofoil;
(3) Stiffened airfoil/hydrofoil with rigid stiffeners;
(4) Stiffened airfoil/hydrofoil with flexible stiffeners;
(5) Stiffened airfoil/hydrofoil with variable cross-section;

The critical buckling pressure is calculated. The NACA0012 airfoil/hydrofoil model is applied here. However, other types of airfoil/hydrofoil is also applicable.

The outline of the paper is presented here. In Section 2, the kinematics of the multi-layered airfoil/hydrofoil structure with stiffeners is set up. The displacement and traction field basis function spaces are defined. In Section 3, the Consistent Orthogonal Basis Function Space for the airfoil/hydrofoil structure is developed. In Section 4, the nonlinear finite element implementation details are presented. In Section 5, the buckling analysis of the hydrofoil under water pressure is presented. In Section 6, the conclusion is given.

## 2. Kinematics

In this section, the kinematics of the airfoil/hydrofoil is set up. The internal surface of the 2-D cross-section of the airfoil/hydrofoil is described by a function as:

$$\begin{aligned} & z = z(y) = z(y_t) \\ & y_t = y/L_c \\ & 0 < y < L_c \end{aligned} \tag{2-1a, b, c}$$

where $L_c$is the chord line length, $y$ is the coordinate of horizontal direction, $z$ is the coordinate of the lift direction.

Given (2-1a) to describe the internal surface of the airfoil/hydrofoil, one can determine its external surface function as:

$$\begin{aligned} z' &= z + \frac{t_w}{\cos(\theta_t)} \\ &= z + t_w\sqrt{1 + (\tan(\theta_t))^2} \\ &= z + t_w\sqrt{1 + (\frac{dz}{dy})^2} \end{aligned} \tag{2-2}$$

where the tangential angle is calculated as:

$$\theta_t = \arctan\left(\frac{dz(y)}{dy}\right) \tag{2-3}$$

and $t_w$ is the thickness of the layer.

The natural coordinates are defined in Fig. 1. In the natural coordinate system, a local origin point $O$ is placed. From the origin point $O$, a radial line is generated that makes an angle of $\theta$ (hoop angle) with $y - axis$. The radial line interacts with the inner surface at point $P$ and with the outer surface at point $P'$.