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An energy-controlling boundary condition for partial wave reflections in the mild slope equation

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ABSTRACT

An energy-controlling technique to actively manage the reflective property of waves from solid boundary is presented. As linear waves propagate through an energy-controlling area, a reduction in wave heights occurs due to energy dissipation, which can be placed under direct control through the imaginary part of the wavenumber and phase velocity. Based on this relationship, the present study investigates a new method to control reflected waves with desired heights in the mild slope equation model. The method is validated through numerical tests for various reflection coefficients and the results confirm the promising use of energy-controlling boundary condition for partial wave reflections.

1. Introduction

Waves propagating from the deep water to the shallow water transform through a series of processes such as diffraction, refraction, shoaling, reflection and wave breaking. The wave transformation process is highly complex, and its prediction is largely dependent on the numerical approach. As numerical solutions inevitably bring about undesirable errors due to the discretization, numerical approaches to minimize errors are required for reliable solutions. In addition to this, appropriate governing equations and boundary conditions are crucial to accurate solutions. This study focuses on boundary conditions related to waves, two of which are most common; open boundary condition and reflection boundary condition. Open boundary condition refers to an artificial treatment at the boundary to help waves propagate out of the computational domain without re-reflection. Reflection boundary condition refers to one that allows perfect or partial wave reflections by structures or topography.

To diminish wave reflections at certain areas of computational domain, the initial concept of sponge layer technique was designed and proposed by Israeli and Orszag [1] and its use was practically demonstrated by Larsen and Dancy [2]. As it provides a convenient and efficient technique to control the wave energy damping, it has been an alternative to the open boundary condition, which may be prone to produce undesirable errors. In practice, however the partial reflection of waves due to structures or topography, even the natural beach with a mild slope, occurs more frequently than the open boundary condition or radiation boundary condition. Thus, this study introduces an energy-controlling method for the practical application of damping layers to generate partial reflection cases based on the linear wave theory. Previously, many studies related to partial reflection have been carried out [3–5] providing accurate calculation results. In the present study, a new method which is more intuitively and easily understood is introduced to manage the wave reflection with arbitrary coefficient at boundaries. The reduction in wave height in energy-controlling areas (i.e., partially-damping layers) is determined, in principle, by the imaginary part of the wavenumber and the wave phase velocity. Based on this relationship, some waves are reflected selectively by adjusting two variables, the width of an energy-controlling area and the imaginary wavenumber in an appropriate way. Although the suggested method must employ an additional energy-controlling area, it is very simple and easy to construct partial-reflection boundary conditions on the theoretical basis. The technique is validated through the mild slope equation model to confirm its accuracy.

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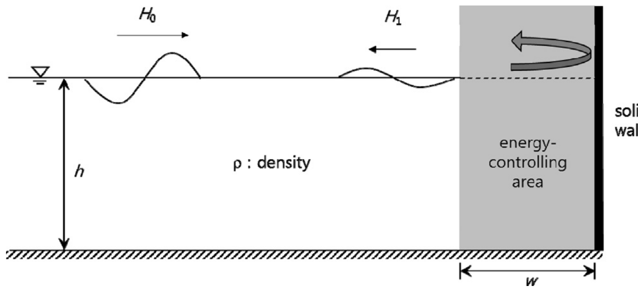

Fig. 1. Schematic diagram for reflected waves by an energy-controlling area.

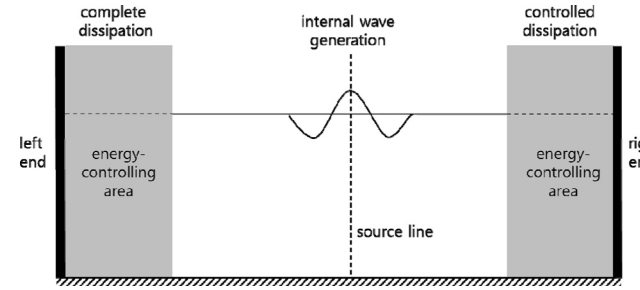

Fig. 2. Computational domain.

2. Derivation of the equations

2.1. Governing equation

The mild slope equation derived for the linear waves is presented as follows [6–8]

$$\frac{\partial Q_x}{\partial t} + c^2 \frac{\partial}{\partial x}(\xi) + f_D Q_x = 0 \quad (1)$$

$$\frac{\partial Q_y}{\partial y} + c^2 \frac{\partial}{\partial y}(\xi) + f_D Q_y = 0 \quad (2)$$

$$\frac{\partial \xi}{\partial t} + \frac{1}{n}\left\{\frac{\partial (nQ_x)}{\partial x} + \frac{\partial (nQ_y)}{\partial y}\right\} = 0 \quad (3)$$

where Q_x and Q_y are the flow rates in the x and y directions, respectively, ξ is the free surface elevation, c is the phase velocity, and n is c_g/c, the ratio of phase speed and group velocity defined as follows.

$$n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh\ 2kh}\right) \quad (4)$$

In Eqs. (1) and (2), f_D is a term added to consider energy dissipation occurring artificially inside the energy-controlling area or due to breaking waves or bottom friction. Therefore, it satisfies the following equation [9,10],

$$\nabla \cdot (c_g E) = -f_D n E \quad (5)$$

where E is the wave energy defined by $\rho g H^2/8$.

Waves propagating over a constant water depth can be represented as

$$H(x) = H_0 \ \exp(ikx) \quad (6)$$

where H_0 is the incident wave height. When energy dissipation of waves occurs, the wavenumber can be expressed by a complex wavenumber of $k_r + ik_i$ so that Eq. (6) can be written as

$$H = H_1 \ \exp(ik_r x) \quad (7)$$

where $H_1(=H_0\, e^{-k_i x})$ is the decreasing wave height as the wave propagates due to energy dissipation, which is related to the imaginary number (k_i) of the wavenumber. The energy dissipation ratio ε_D related to f_D is calculated as follows.

$$\varepsilon_D = \frac{d}{dx}(c_g E) = c_g \left(2\rho g \frac{H_1}{8}\frac{dH_1}{dx}\right) = -2 c_g k_i E \quad (8)$$

From Eqs. (5) and (8), f_D can be expressed as

$$f_D = 2k_i c \quad (9)$$

2.2. Determination of k_i

As shown in Fig. 1, incident waves with the wave height of H_0 propagate through the energy-controlling area with a width, w. Incident waves are reflected completely from the wall and returning back to the previous state passing through the energy-controlling area again in opposite direction with the wave height of H_1. In such case, k_i value can be determined by Eq. (7) as follows

$$k_i = -\frac{\ln(H_1/H_0)}{2w} = -\frac{\ln C_r}{2w} \quad (10)$$

where C_r is the reflection coefficient. Using Eq. (10), k_i value can be tuned artificially to produce the desired reflection coefficient under the given width of an energy-controlling area. In this manner, partially reflected waves can be realized, promising the use of energy-controlling method for partial reflection of waves. It is worthy to note that the limiting condition of $C_r = 0$ is theoretically in accordance with $k_i = +\infty$, which can be justified considering no wave reflection resulting from infinite(i.e., complete) dissipation of energy.

3. Verification of the equation

Using a numerical solution, the equation derived in the previous section was verified. The numerical solution was calculated through the application of the finite difference method that employed staggered grids [10] to Eqs. (1)–(3). The discretized equations of the one-dimensional numerical model are as follows:

$$\xi_i^{t+\Delta t} = \xi_i^t - \frac{\Delta t}{\Delta x}\left[(nQ_x)_{i+1}^t - (nQ_x)_i^t\right] \quad (11)$$

$$(Q_x)_i^{t+\Delta t} = (Q_x)_i^t - \frac{\Delta t}{\Delta x} c_i^2 \left[\xi_i^t - \xi_{i-1}^t\right] + \Delta t f_D \quad (12)$$

where ξ is the displacement of the free surface, Q is the flow rate and g is the gravity acceleration. In the numerical tank for simulation, an internal wave maker was utilized [11,12] at the center of domain while an energy-controlling area was deployed at each end with different purposes as shown in Fig. 2. No re-reflected wave was expected to be generated at the left end since typical values for complete energy dissipation were used for f_D. At the right end, meanwhile, k_i values determined by Eq. (10) were applied to provide the partial reflection condition with a desired coefficient. The water depth was set to 4 m and the wave period was determined to such a value that keeps the relative water depth as 0.1π.

Fig. 3 depicts simulation results of instantaneous surface elevations at different phases in the case of $C_r = 0.5$. Surface elevation at $t = 40.5T$ creates maximum excursion of the free surface with a normalized amplitude of 1.5, while those at $t = 40.25T$ and $40.75T$ result minimum. It is also clearly observed the energy-controlling effects in the shaded area on the right end. Fig. 4 shows the calculated reflection coefficients with errors for different widths of the energy-controlling areas on the right end. It appears that there is a discrepancy for the low reflection coefficients while good agreement is observed for larger values. The calculated results show obvious overestimation in reflection coefficients than the target ones, and this develops as lower reflection coefficients with limited discrepancy. However, the calculated results are improved as the width of an energy-controlling area increases. For larger coeffi-