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Stochastic analysis of the second-order hydrodynamic quantities for offshore structures

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ABSTRACT

The present study considers the prediction of extreme values of the second-order hydrodynamic parameters related to offshore structures in waves, where the application of Gaussian distribution is not valid. Particularly, this study focuses on a characteristic function approach in the frequency domain to estimate the probability distribution of the second-order quantities, and the results are compared with direct simulations in the time domain. The stochastic behaviors of the second-order hydrodynamic quantities are investigated with the characteristic function approach, which involves eigenvalue analyses of Hermitian kernels constructed with quadratic transfer functions. Three different second-order responses are considered: the springing responses of TLP tendons representative of the sum-frequency problem, the slow-drift motions of a semi-submersible platform moored in waves as a representative of the difference-frequency problem, and the wave run-up around a vertical column for regular and irregular waves. The applicability of the present approach in predicting extreme values is assessed by comparing the results with the values obtained from time-domain signals.

1. Introduction

The offshore oil and gas industry has grown up significantly over the last decades, and the relevant technologies are mature enough to allow the developers to advance to ultra-deep sea. For example, the ‘Perdido’ spar has been in operation since 2010 from a water depth of 2450 m in the Gulf of Mexico, and the ‘Big Foot’ Tension Leg Platform (TLP) is planning to produce its first oil in 2016 from a water depth of 1585 m. As the installation water depth increases, the probability of failure of the offshore structure may increase, because it needs to sustain a possibly larger payload and has longer and more complex mooring lines and risers. Also, offshore structures in deep sea are likely to encounter more severe sea states. To reflect this high probability of failure in deep water, the design load cases have become increasingly conservative, and now the return period of the environmental condition used for seakeeping analyses reaches up to 10,000 years.

When an offshore structure is subjected to such severe environmental conditions, the hydrodynamic responses of the platform or the lines can no longer be predicted with the linear theory since apparent nonlinear phenomena may be significant or may even dominate the responses. And in most cases, the discrepancies between the linear theory and the nonlinear responses are well compensated by including second-order terms in the analyses. Representative examples of these nonlinear hydrodynamic responses which can be explained by the second-order theory are the springing responses of TLP tendons, the slow-drift motions of moored offshore platforms, and the wave elevation around a vertical column. The springing responses refer to high-frequency resonant heave or pitch motions of the TLP and the corresponding fluctuations that occur in the tendon tensions. Because the natural period of heave motions of TLP are generally around 3 s, which is far lower than the wave periods, second-order sum-frequency wave loads are adopted to explain the resonant behaviors. The slow-drift motion of a moored offshore platform is also a resonant response induced mostly by second-order wave loads. In this case, because moored floating platforms have very high natural periods in horizontal motions, the resonance is explained by including the second-order difference-frequency wave loads. The wave elevation around a vertical column is an important parameter that determines the air gap of an offshore platform; however, it was proved both theoretically and experimentally by Kriebel [1] that a significant underestimation could be made if the nonlinear components of the wave elevations are not considered. According to the study, average discrepancies between the experimental and the theoreti-

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cal results reduced to one-quarter when including the second-order components of the wave elevations.

The aforementioned second-order hydrodynamic quantities can be mathematically modeled as a two-term Volterra series. Because this two-term Volterra series is a nonlinear function, a Gaussian stochastic input, which in hydrodynamics is mostly a linear irregular wave train, does not lead to a Gaussian stochastic output. Hence, a number of studies dealt with determining the probability distribution of the two-term Volterra series model. Kac and [2] and Bedrosian and Rice [3] first developed the probabilistic distribution of this model in the communication field. Their approach was to calculate the characteristic function of the random variable by solving the eigenvalues of the integral equation constructed with the corresponding kernel of the system. Later, Neal [4] first applied this theory to the ocean engineering field, deriving the integral equation for the probability distribution of the second-order hydrodynamic forces. After that, many researchers applied the theory to second-order hydrodynamic problems. Naess [5] and Kim [6] derived the probability distribution of the second-order difference-frequency wave loads explicitly by reducing the inverse Fourier transforming of the characteristic function to a series of Cauchy integrals. Application of the theory to the sum-frequency springing responses of TLPs was made later by Kim and Yue [7], Naess and Ness [8], Naess [9], and Eatock Taylor and Kernot [10].

Most of the studies described above dealt with the probability distribution of the second-order quantity. To predict the extremes of the quantity in a certain duration, however, the exceedance probability distribution of peak values needs to be estimated. For the slow-drift forces and motions, Naess [11,12] investigated the extreme-value behavior by estimating the up-crossing frequencies asymptotically. However, the equation of motion was assumed to be completely linear, which is rather oversimplified as nonlinear restoring and damping significantly affect slow-drift motions. Later, Naess and Johnsen [13] and Naess, Gaidai, and Teigen [14] dealt with the statistics of the slow-drift responses from a nonlinear equation of motion, but this subject is still under investigation owing to its complex nonlinearities. In the case of springing responses, extreme-value statistics have not been considered as much because the springing response is more relevant to the fatigue damage to tendons. However, the maximum response of springing should also be carefully examined to ensure design safety. The second-order wave elevation around a vertical cylinder has been investigated by many researchers. However, most of these studies focused on analytical or numerical computations of wave elevation (Kriebel, [1]; Kriebel, [15]; Eatock Taylor and Huang [16]) and not its stochastic behaviors.

In this study, the stochastic behaviors of the second-order hydrodynamic quantities and their extreme-value distributions are reinvestigated. The probability distributions of the second-order quantities are calculated based on the characteristic function approach, and the extreme-value statistics are estimated assuming a narrow bandwidth. The aforementioned three second-order hydrodynamic quantities are considered: the springing responses of TLP tendons, the slow-drift motions of moored offshore platforms, and the wave elevation around a vertical column. The applicability of the present approach in predicting the extreme values for each of the second-order quantities is assessed by comparing the results with direct simulations in the time-domain.

2. Mathematical background

2.1. Probability distribution of two-term volterra series

As mentioned briefly in the introduction, the second-order hydrodynamic quantity is expressed as a two-term Volterra series which is a nonlinear function of an input signal. Hence, even though the input signal is Gaussian, the output signal does not necessarily follow the same stochastic model. However, the probability distribution of the second-order response can be estimated semi-analytically using the characteristic function method as the stochastic model of the input signal is known and the mathematical model for the response is well defined.

Let $Y(t)$ be a nonlinear hydrodynamic response at a certain point to unidirectional random waves $X(t)$. Then $Y(t)$ can be represented as a function of $X(t)$ as

$$Y(t) = F\left(X(t)\right), \quad \left\{X(t)| -\infty < t < \infty\right\} \tag{1}$$

where $F(x)$ is the corresponding nonlinear function of $X(t)$. If $F(x)$ is continuous, Eq. (1) can be expanded in a functional power series:

$$Y(t) = g_0 + \int_{-\infty}^{\infty} g_1(t, t_1)X(t_1)dt_1 + \cdots + \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g_n (t, \cdots, t_n)X(t_1)\cdots X(t_n)dt_1\cdots dt_n + \cdots \tag{2}$$

where g_i indicates integral kernels for each order. Assuming the nonlinear system is causal and time-invariant, and limiting the analysis to include excitation effects up to second order, Eq. (2) becomes a two-term Volterra series as follows:

$$Y(t) = Y_0 + Y_1(t) + Y_2(t) = h_0 + \int_0^{\infty} h_1(t_1)X(t-t_1)dt_1 + \int_0^{\infty}\int_0^{\infty} h_2(t_1,\ t_2)X(t-t_1)X(t-t_2)dt_1dt_2 \tag{3}$$

In Eq. (3), $Y_n(t)$ indicates the n-th order component of total response $Y(t)$, and $h_1(t_1)$ and $h_2(t_1,\ t_2)$ are linear and quadratic impulse response functions, respectively. The zero-th order term Y_0 is hereby neglected since it is not a stochastic variable.

The response $Y(t)$ can also be expressed alternately with linear and quadratic transfer functions obtained in the frequency domain. Let the random wave $X(t)$ be expressed as a sum of sinusoidal component waves, then the equivalent expressions of $Y_1(t)$ and $Y_2(t)$ are obtained as follows:

$$X(t) = Re\left[\sum_{j=1}^{\infty} A_j e^{i\omega_j t}\right] \tag{4}$$

$$Y_1(t) = \int_0^{\infty} h_1(t_1)X(t-t_1)dt_1 = Re\left[\sum_{j=1}^{\infty} A_j H_1(\omega_j) e^{i\omega_j t}\right] \tag{5}$$

$$Y_2(t) = \int_0^{\infty}\int_0^{\infty} h_2(t_1,\ t_2)X(t-t_1)X(t-t_2)dt_1dt_2 = Re\left[\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} A_jA_kH_2(\omega_j,\omega_k)e^{i(\omega_j+\omega_k)t} + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} A_jA_k^*H_2(\omega_j,-\omega_k)e^{i(\omega_j-\omega_k)t}\right] \tag{6}$$

In the equations above, ω_j and A_j indicate the frequency and the complex amplitude of the j-th wave component, respectively, and $H_1(\omega_j)$ and $H_2(\omega_j,\ \omega_k)$ represent the linear transfer function (LTF) and the quadratic transfer function (QTF), respectively. Upper script $()^*$ indicates the complex conjugate of a quantity. From Eqs. (5) and (6), the frequency-domain transfer functions are derived as the Fourier transforms of impulse response functions as follows:

$$H_1(\omega) = \int_{-\infty}^{\infty} h_1(\tau)e^{-i\omega\tau}d\tau \tag{7}$$

$$H_2(\omega_j,\ \omega_k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)e^{-i(\omega_j\tau_1+\omega_k\tau_2)}d\tau_1d\tau_2 \tag{8}$$