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Assessment of uncertainty in environmental contours due to parametric uncertainty in models of the dependence structure between metocean variables

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ABSTRACT

A key factor for computing environmental contours is the appropriate modeling of the dependence structure among the environmental variables. It is known that all the information on the dependence structure of a set of random variables is contained in the copulas that define their multivariate probability distribution. Provided that copula parameters are estimated by means of statistical inference using observations, recordings, numerical or historical data, uncertainty is unavoidably introduced in their estimates. Parametric uncertainty in the copulas parameters then introduces uncertainty in the environmental contours. This study deals with the assessment of uncertainty in environmental contours due to parametric uncertainty in the copula models that define the dependence structure of the environmental variables. A point estimation approach is adopted to estimate the statistics of the uncertain coordinates of the environmental contours considering they are given in terms of inverse functions of conditional copulas. A case study is reported using copulas models estimated from storm hindcast data for the Gulf of Mexico. Uncertainty in environmental contours of significant wave height, peak period and wind speed is assessed. The accuracy of the point estimation of the mean and variance of the contour coordinates is validated based on Monte Carlo simulations. A parametric study shows the manner in which greater parametric uncertainty induces larger variability in the environmental contours. The influence of parametric uncertainty for different degrees of association is also analyzed. The results indicate that variability between contours considering parametric uncertainty can be meaningful.

1. Introduction

Environmental contours are a useful and practical means to characterize multivariate natural hazards for structural design purposes. They define combinations of possible values of environmental variables that should be considered for finding the maximum system response associated with a given exceeding probability or return period [1,2]. Computational techniques for estimation of environmental contours and applications in offshore, earthquake and wind engineering can be found in [3–12]. The computation of environmental contours requires the description of the joint probability distribution of the environmental variables. For instance, building environmental contours of extreme sea states for offshore applications requires the description of the multivariate distribution of metocean variables such as significant or maximum wave height, peak spectral period, wind velocity, or current velocity. Provided the Rosenblatt transformation is applied for computing environmental contours, the joint distribution of the environmental variables can be described in terms of a set of conditional distributions. For instance, in offshore engineering applications, the marginal distribution can be estimated for the significant wave height (H_S), and then the distribution of the peak period (T_P) can be modeled conditionally on the wave height [13]. A large amount of data is required for estimation of the parameters of conditional distributions. Such requirement increases greatly with the problem dimension, making more difficult its application in practical cases without having to introduce some independence assumptions when more than three or four variables are considered. Another approach is to estimate the marginal distributions and the linear correlation matrix to assemble the joint distribution using the Nataf model [14,15].

A key factor for developing environmental contours is the appropriate modeling of the dependence structure among the

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environmental variables which is present in the multivariate probability distribution. Recent studies have shown that the statistical dependence between environmental variables has a significant influence on the contours [9,16]. Focusing on a rigorous modeling of the dependence structure, Montes-Iturrizaga and Heredia-Zavoni have proposed procedures for the use of copulas to compute environmental contours [16,17]. Copulas are functions that fully define the multivariate distribution of a set of random variables [18]. It can be shown that multivariate distributions can be expressed in terms of the marginal distributions and the density of the copula, where all the information on the dependence structure of the random variables is contained. An advantage of the copula approach is that the development of an appropriate model for the dependence structure can be carried independently from the particular choices of marginal distributions of the random variables. Using copula theory, it has become clear that assembling multivariate distributions using the Nataf model yields the dependence structure of a Gaussian copula [19].

There are several methods for estimating the parameters of a copula. Measures of association such as Kendall's τ or Spearman's ρ can be used as estimators for one-parameter copulas [20]. The more general method of maximum pseudo-likelihood (MPL) is available for multidimensional copula parameters [21]. Formulations for computing the statistics of the copula parameter estimators are also available. Provided that copula parameters are estimated by means of statistical inference using observations, recordings, numerical or historical data, uncertainty is unavoidably introduced in their estimates. Parameter uncertainty is therefore involved in the copulas that model the dependence structure among the environmental variables. Since environmental contours rely on the characterization of the dependence among environmental variables, uncertainty is also introduced in their computation. This study deals with the assessment of the uncertainty in environmental contours due to parametric uncertainty in the modeling of the dependence structure between the environmental variables.

We first introduce some basic concepts on modeling statistical dependence using copulas and discuss the formulation for the computation of environmental contours in terms of copulas. The point-estimation method adopted to estimate the statistics of the coordinates of the environmental contours is presented next. The effects of parametric uncertainty on environmental contours of significant wave height vs peak period and significant wave height vs wind speed are then examined using storm hindcast data from the Gulf of Mexico. Validation of the point estimation method using Monte Carlo simulations is analyzed and results of parametric analyses are then given focusing also on the influence of parametric uncertainty considering different degrees of association between the metocean variables. A summary of the work and concluding remarks are given at the end.

2. Modeling dependence for environmental contours

Consider an engineering system exposed to a natural event whose occurrence in time is uncertain. Let $\mathbf{X}$ denote the vector of uncertain environmental variables that characterize such natural event for assessing the system response. The environmental contours are defined by the values of the environmental variables $\mathbf{x}$ in the physical space corresponding to the values of vector $\mathbf{z}$ in a reduced space where $|\mathbf{z}| = \beta$ [1]. In other words, the environmental contour is the image of environmental variables in the physical space corresponding to a d-dimensional hypersphere of radius β in the reduced space. The mapping from the physical space of $\mathbf{X} = (X_1, \cdots, X_d)$ into the reduced space of independent standard normal random variables $\mathbf{Z} = (Z_1, \ldots, Z_d)$, is performed using the Rosenblatt isoprobabilistic transformation [22]. The inverse of this transformation is given by,

$$
\begin{aligned}
x_1 &= F_1^{-1}(e_1) \\
x_2 &= F_{2|1}^{-1}(e_2|x_1) \\
x_d &= F_{d|1,\cdots,d-1}^{-1}(e_d|x_1, , \cdots, x_{d-1})
\end{aligned}
\tag{1}
$$

where $F_i(x_i)$ is the marginal cumulative distribution function of X_i, $F_{i|1,\cdots,i-1}(x_i|x_1, \cdots x_{i-1})$. is the conditional distribution of X_i given $X_1 = x_1, \cdots, X_{i-1} = x_{i-1}$, and $e_j = \Phi\left(z_j\right), j = 1, \cdots, d$, Φ being the standard normal distribution function. Let p_F denote the exceedance probability of the system response to the natural event characterized by $\mathbf{X}$ and assume the occurrence of such event is modeled as a Poisson process with mean annual rate λ_E; the annual exceeding probability is then $p_a = 1 - \exp(-\lambda_E p_F)$. Considering that $\beta = -\Phi^{-1}(p_F)$ and that the return period $T_R = 1/p_a$, the radius of the hypersphere is defined by $\beta = -\Phi^{-1}[-\lambda_E^{-1}\ln(1 - 1/T_R)]$. Only environmental variables are considered to be uncertain and the system response is a deterministic function. The environmental contours developed can be used to search for the maximum system response associated with a return period or exceedance probability for design purposes.

Developing environmental contours requires a joint probability description of the environmental variables. Copulas are functions which define the multivariate probability distribution of a random vector or a set of random variables, and, as such, characterize the dependence structure of the random variables. They are functions that couple multivariate distribution functions to their marginal distributions. Copulas have uniform one-dimensional margins on the interval $[0, 1]$ and are invariant under monotone increasing transformations of the marginal [18]. Let us consider vector $\mathbf{X}$ of random variables with marginal distribution functions $F_i(x_i)$, $i = 1, \cdots, d$. The set of transformations $U_i = F_i(X_i)$ define a dependent, uniformly distributed vector of random variables $\mathrm{U} = (U_1, \cdots, U_d)$ on $[0, 1]^d$. If the functions $F_i(x_i)$ are continuous, then, the joint distribution function of $\mathbf{X}$ can be expressed as [18]

$$
F(\mathrm{x}) = C_\theta(F_1(x_1), \ldots, F_d(x_d)) = C_\theta(u_1, \cdots, u_d) \tag{2}
$$

where $C_\theta(\boldsymbol{u})$ is the copula of the distribution, $C_\theta : [0, 1]^d \to [0, 1]$, $\mathbf{u} = (u_1, \ldots, u_d)$, and θ is the vector of copula parameters. The copula C_θ is also a joint distribution function of vector $\mathbf{U}$. Eq. (2) is known as Sklar's theorem [18]. Furthermore, copula $C_\theta(\boldsymbol{u})$ is defined by

$$
C_\theta(\boldsymbol{u}) = F\left(F_1^{-1}(u_1), \ldots, F_d^{-1}(u_d)\right) \tag{3}
$$

and the corresponding copula density is given by,

$$
c_\theta(\boldsymbol{u}) = \frac{\partial^d C_\theta(u_1, \ldots, u_d)}{\partial u_1 \ldots \partial u_d} \tag{4}
$$

The joint probability density function of $\mathbf{X}$, $f_{\mathbf{X}}(\boldsymbol{x}) = f_{\mathbf{X}}(x_1, \ldots, x_d)$, is given by [23]:

$$
\boldsymbol{f}_{\mathbf{X}}(x_1, \ldots, x_d) = c_\theta\{F_1(x_1), , \ldots, F_d(x_d)\} \prod_{i=1}^{d} f_i(x_i) \tag{5}
$$

where $f_i(x_i)$ is the marginal probability density function of X_i. Eq. (5) clearly shows that the joint probability density is assembled by the collection of marginal distributions and by the copula density which retains all the information on the dependence structure of the random variables. In general, the conditional marginal distributions of $\mathbf{X}$ can be calculated from (see, e.g. [19,24])

$$
F_{i|1,\cdots,i-1}(x_i|x_1, \cdots, x_{i-1}) = C_{\theta i|1,\cdots,i-1}(u_i|u_1, \cdots, u_{i-1}) \tag{6}
$$