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# Uncertainty of full-scale manoeuvring trial results estimated using a simulation model

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ABSTRACT

This paper describes how to estimate the uncertainty of manoeuvring sea trial results without performing repeated tests using only a simulation model. The approach is based on the Monte Carlo method of uncertainty propagation. Moreover, the global sensitivity analysis procedure based on variance decomposition is described. As an example, the method is applied to estimate the uncertainty of 10°/10° zigzag overshoot angles and a 20° turning circle advance and tactical diameter for a small research vessel. The estimated uncertainty is compared with corresponding experimental uncertainty assessed from repeated tests. The method can be useful for validation studies and other studies that involve the uncertainty of sea trial results.



## 1. Introduction

Currently, ship manoeuvring simulation models are widely applied for training, standardization and engineering purposes. However, recent studies [1] have shown a large scatter in the results predicted by models developed by different organizations. This indicates high demand for the development of validation techniques, as well as for the improvement of ship simulation models. The ITTC Manoeuvring Committee indicates the importance and a lack of validation activities for ship simulators [2,3]. Validation is performed via comparison of predictions made by a simulator with experimental results for identical trials. Full-scale or model-scale experiments are used to obtain a benchmark for validation. However, model-scale experiments are prone to scale effects; therefore, full-scale experiments are preferable. Evaluation of the uncertainty of experimental results is an important part of validation. The ITTC issued a recommended procedure for the uncertainty analysis of free running model tests [4] that outlines the main sources of uncertainty in manoeuvring experiments. They use a combination of three approaches to obtain the combined uncertainty: measurement uncertainty analysis, repeatability analysis and uncertainty propagation analysis. The uncertainty propagation analysis is based on the Taylor series method. According to the method, the uncertainty of the experimental result due to some input factor is equal to the product of the uncertainty of this factor and the so-called uncertainty magnification factor (UMF). The UMF is a linear local absolute sensitivity coefficient and can be numerically estimated using a simulation model. The total combined uncertainty is calculated as the root summed squared of the individual uncertainty contributions. Although this approach can be partly applied to full-scale tests, some features cause a significant difference between full-scale tests and model-scale tests. The price of repetitions is very high for full-scale tests; therefore, the repetitions are rarely performed. Moreover, sea trial results are influenced by environmental effects, whose contribution to the resulting uncertainty is sometimes dominating. These effects are represented by two or more independent factors (such as current speed and direction, or wave height, period and direction) with strong interaction. Therefore, the combined result uncertainty cannot be estimated using the UMFs.

In this paper, we consider an alternative approach to uncertainty propagation based on the Monte Carlo method [5,6]. The Monte Carlo method is more flexible and suitable for highly nonlinear systems and interconnected input factors. The Monte Carlo method was previously used in manoeuvring to propagate the uncertainty of force measurements in captive tests to the final uncertainty of manoeuvring indices (overshoot angles) [7,8]. We apply the method to estimate the uncertainty of repeated tests and compare it with the experimentally determined uncertainty. We also describe the global sensitivity analysis based on variance decomposition and apply it to estimate the contribution of the input factors to the total uncertainty. Thus, the main goal of the paper is to demonstrate how to estimate uncertainty of full-scale manoeu-

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**Nomenclature**

| | |
|---|---|
| $\sigma$ | Standard deviation |
| CDF | Cumulative distribution function |
| $N(\mu, \sigma)$ | Normal distribution with the mean $\mu$ and the standard deviation $\sigma$ |
| OA | Overshoot angle |
| PDF | Probability distribution function |
| $U(\xi_1, \xi_2)$ | Uniform distribution, with the lower and upper borders $\xi_1$ and $\xi_2$ |
| $U_{95}$ | Expanded uncertainty, with 95% confidence |
| UMF | Uncertainty magnification factor |

vring tests in practice, without actually performing repeated tests. Therefore, we do not consider some possible sources of uncertainty which are negligible, while focusing on more important ones. We emphasize also that all the sensitivity coefficients obtained are specific for the case vessel and prescribed conditions. However, the same algorithm can be applied to any other vessel.

The paper is organized as follows. Section 2 describes the Monte Carlo method of uncertainty propagation. Section 3 describes how to estimate the contribution of the input factors to the total uncertainty of the result using graphical analysis and variance decomposition. Section 4 describes how to use the Monte Carlo method to estimate the uncertainty of manoeuvring trial results. Section 5 presents an example of the application of the analysis to estimate the uncertainty of 10°/10° zigzag and 20° turning circle test results. Section 6 contains discussion and conclusions.

## 2. Monte Carlo method

Consider the result $Y$ predicted by a simulation model. The result depends on the set of uncertain input factors $\boldsymbol{X}$:

$$Y = f(\boldsymbol{X}) \tag{1}$$

Each of the input factors in $\boldsymbol{X}$ has an associated known probability distribution. The goal of the Monte Carlo propagation method is to estimate the uncertainty of $Y$ due to the uncertainty of $\boldsymbol{X}$. The following algorithm describes the procedure:

1.) Generate a matrix $\boldsymbol{A}$ with numbers distributed randomly on [0,1], with $N$ rows and $k$ columns, where $N$ is a sufficiently large number, k is the number of input factors in $\boldsymbol{X}$. $N$ defines the total number of simulations.

2.) Apply the corresponding inverse cumulative distribution function to the samples of each column and compose a new matrix $\boldsymbol{X}$ from the resulting numbers:

$$x_{ji} = CDF_i^{-1}\left(a_{ji}\right) \tag{2}$$

3.) Now, each row of the matrix $\boldsymbol{X}$ contains a set of input parameters for (1). Perform the simulations for each row of $\boldsymbol{X}$ and calculate the array of results $\boldsymbol{Y}$ according to (1).

4.) According to the central limit theorem [9], the result $\boldsymbol{Y}$ is distributed approximately normally if it is not dominated by a single input factor. Therefore, calculate the standard deviation $\sigma$ of $\boldsymbol{Y}$ and then calculate expanded uncertainty $U_{95}$ by multiplying $\sigma$ by the coverage factor 2. In some cases, the resulting distribution of $\boldsymbol{Y}$ is not close to a normal distribution. Then, to find the 95% confidence interval, build the empirical CDF of $\boldsymbol{Y}$ and find the lower and the upper border of the confidence interval as the argument of the function, where it equals 0.025 and 0.975, respectively.

Thus, the Monte Carlo method provides the uncertainty of the model output due to the uncertainty of the input factors $\boldsymbol{X}$. However, the method does not provide information regarding the contribution of each individual input factor to the total uncertainty.

## 3. Global sensitivity analysis

In the Taylor series method of uncertainty propagation, the individual contributions of the input factors' uncertainty are calculated as a part of the analysis. In the Monte Carlo method, it is not possible to say directly which input parameters make the main contribution to result uncertainty. However, this knowledge is very useful. It increases confidence in the uncertainty analysis and helps to detect faults or to improve the experiment.

We consider two methods to assess the relative importance of the input factors. The first method is graphical. According to the method, one should plot the result $Y$ versus the input factor $X_i$ (this will be illustrated in subsection 5.5, see Fig. 6). If the result changes for different values of the input factor, or, in other words, a pattern is observed, the parameter is important. The stronger the pattern is, the more important the parameter is. The method is simple and does not demand additional simulations. However, it does not give any objective quantitative measure of the factor's importance, and it is not suitable for studying the joint effects of several interacting factors.

The second method is based on variance decomposition. The further description of the method closely follows [10,11]. The method uses the concept of a sensitivity index as the measure of factors' importance. There are first-order, joint and total effect sensitivity indices. The first-order sensitivity index $S_i$ and the total effect $S_{Ti}$ of the factor $X_i$ are defined correspondingly as:

$$S_i = \frac{V_{X_i}(E_{\boldsymbol{X}_{\sim i}}(Y|X_i))}{V(Y)} \tag{3}$$

$$S_{Ti} = \frac{E_{\boldsymbol{X}_{\sim i}}(V_{X_i}(Y|\boldsymbol{X}_{\sim i}))}{V(Y)} = 1 - \frac{V_{\boldsymbol{X}_{\sim i}}(E_{X_i}(Y|\boldsymbol{X}_{\sim i}))}{V(Y)} \tag{4}$$

where $V_{Xi}(\ldots), E_{Xi}(\ldots)$ is the variance and the mean, respectively, of the argument $(\ldots)$ taken over $X_i$; $V_{X\sim i}(\ldots), E_{X\sim i}(\ldots)$ is the variance or mean of the argument $(\ldots)$ taken over all factors but $X_i$; $V(Y)$ is unconditional variance. Thus, $S_i$ is the expected relative reduction in variance $V(Y)$ that would be obtained if $X_i$ could be fixed; $S_{Ti}$ is the expected relative variance that would be left if all factors but $X_i$ could be fixed. The joint sensitivity indices are defined by analogy, for instance, for two factors:

$$S^c_{i,j} = \frac{V_{\boldsymbol{X}_{ij}}(E_{\boldsymbol{X}_{\sim ij}}(Y|\boldsymbol{X}_{ij}))}{V(Y)} \tag{5}$$

The higher-order effects (interactions) are defined as the residual component as:

$$S_{ij} = S^c_{i,j} - S_i - S_j \tag{6}$$

In fact, the unconditional variance can be decomposed as the sum of the first-order and higher-order effects:

$$1 = \frac{V(Y)}{V(Y)} = \sum_i S_i + \sum_i \sum_{ji} S_{ij} + \sum_i \sum_{ji} \sum_{lj} S_{ijl} + \ldots + S_{12\ldots k} \tag{7}$$

The total effect of the factor $X_i$ contains all terms in (7) that involve this factor, for instance:

$$S_{T1} = S_1 + S_{12} + S_{13} + S_{123} + \ldots \tag{8}$$

To sum up the method, we list important properties of the sensitivity indices: