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Proof of the equivalence of Tanizawa–Berkvens' and Cointe–van Daalen's formulations for the time derivative of the velocity potential for non-linear potential flow solvers

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ABSTRACT

Dealing with freely-floating bodies in the framework of non-linear potential flow theory may require solving Laplace's equation for the time derivative of the velocity potential. At present, there are two competing formulations for the body boundary condition. The first one was derived by Cointe [1] in 2D. It was later extended to 3D by van Daalen [2]. The second formulation was derived by Tanizawa [3] in 2D. It was extended to 3D by Berkvens [4]. In this paper, a proof is given that the Cointe–van Daalen's and the Tanizawa–Berkvens' formulations are equivalent. It leads to a simplified version of Cointe–van Daalen's formulation. The formulation is validated against the analytical solution for a moving sphere in an unbounded water domain.

1. Introduction

The first breakthrough for the simulation of non-linear waves in time domain came from the Mixed Euler Lagrande (MEL) method, introduced by Longuet-Higgins and Cokelet [5]. Empowered by this approach, many numerical wave tanks (NWT) were developed, to simulate non-linear wave propagation [6], plunging waves [7,8], overturning waves [9] in 2D or 3D. Simulations of wave diffraction on bodies with fixed or prescribed motions were also possible [10,11]. The accurate evaluation of the time derivative of the velocity potential on the body, giving the hydrodynamic pressure, could be done with a finite difference scheme in this particular case. However this scheme leads to numerical instabilities when considering freely moving bodies [1,12] with explicit time-stepping schemes. Moreover, since the hydrodynamic pressure was needed to solve the body motion equations, a method solving the mechanical and hydrodynamic problems simultaneously was required.

Four methods were proposed along the years in order to cope with this difficulty. The modal decomposition was first developed by Vinje and Brevig [13] and further implemented by Cointe [14]. A second method is the iterative method, used by Sen [15] and Cao [16], and based on a predictor-corrector loop to converge on the body accelerations. The indirect method was introduced by Wu and Eatock Taylor [17] and used by Kashiwagi [11]. It is the only method that does not require solving the Laplace equation for the time derivative of the velocity potential. However it solves directly the body motions without calculating the hydrodynamic force on the body. The last method was introduced simultaneously by Tanizawa [3] and van Daalen [2] and is called the Implicit Boundary method. Further details on these methods can be found in the previously mentioned references, but also in several reviews [18,12,19]. Except for the indirect one, the three other methods require solving the Laplace equation for the time derivative of the velocity potential, the main difficulty lying in its body boundary condition.

Two expressions were given for this body boundary condition. One was proposed by Cointe [14], first in 2D and later extended by van Daalen [2] in 3D. The second was given by Tanizawa [3] in 2D, and Berkvens [4] in 3D. Even if both expressions are implemented in several numerical flow solvers, their equivalence has not been proven yet. This paper will thus be dedicated to prove that these expressions are

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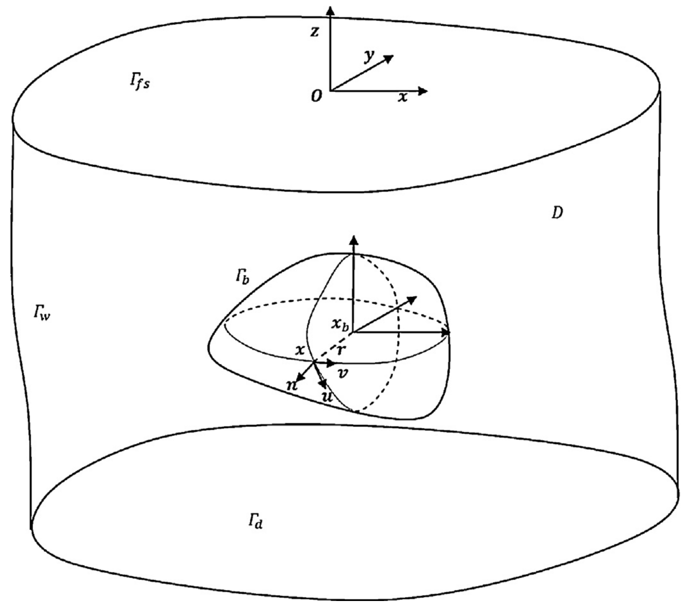

Fig. 1. Domain definition: boundaries and reference frames.

equivalent. The four expressions were moreover developed using different notations and reference systems that are not always explicited in the references. We thus provide here with a common ground for the comparison of those expressions.

The developments of both expressions are first provided, in 2D and 3D. The equivalence is then demonstrated, leading to a new unified expression, simplifying Cointe–van Daalen ones by suppressing second order derivatives with already known variables, and thus ensuring a more accurate estimation of the Body Boundary condition. A simple analytical case of a sphere in prescribed motion in an unbounded water domain is finally applied to ensure that the body condition is correct.

2. Theory

2.1. Potential flow theory

Assuming a fluid to be incompressible and inviscid with irrotational flow, its flow velocity derives from a velocity potential ϕ which satisfies the Laplace Eq. [20]:

$$\nabla^2\phi(x, y, z, t) = 0 \tag{1}$$

in the fluid domain, D. The boundary of the fluid domain is $\partial D = \Gamma = \Gamma_{fs} \cup \Gamma_b \cup \Gamma_w \cup \Gamma_d$, see Fig. 1.

In the general case, without forward speed, it can be shown [21] that the velocity potential is the solution of the following boundary value problem:

$$\begin{cases} \nabla^2\phi = 0 & \text{in the fluid domain} \quad D \\ \dfrac{\partial\phi}{\partial t} = -g\eta - \dfrac{1}{2}\nabla\phi\cdot\nabla\phi & \text{on the free surface,} \quad \Gamma_{fs} \\ \dfrac{\partial\eta}{\partial t} = \dfrac{\partial\phi}{\partial z} - \nabla\phi\cdot\nabla\eta & \text{on the free surface,} \quad \Gamma_{fs} \\ \dfrac{\partial\phi}{\partial n} = \mathbf{V}_b\cdot\mathbf{n} & \text{on the body,} \quad \Gamma_b \\ \dfrac{\partial\phi}{\partial n} = 0 & \text{on the seabed,} \quad \Gamma_d \\ \phi \longrightarrow 0 & \text{on boundaries far from the body,} \quad \Gamma_w \end{cases} \tag{2}$$

The free-surface elevation is denoted by the single-valued variable η. g is the gravitational constant, $\mathbf{V}_b$ the body velocity and $\mathbf{n}$ the normal vector pointing outsides from the fluid.

Using Green's Second Identity, it can be shown that the resolution of the 3D Laplace equation in the fluid domain can be reduced to a boundary value problem (BVP) for the velocity potential. For any point $\mathbf{x} \in D$,

$$\alpha(\mathbf{x})\phi(\mathbf{x}) = \int_\Gamma \left[\frac{\partial\phi}{\partial n}(\mathbf{x}_l)G(\mathbf{x}, \mathbf{x}_l) - \phi(\mathbf{x}_l)\frac{\partial G}{\partial n}(\mathbf{x}, \mathbf{x}_l)\right] d\Gamma \tag{3}$$

where $\alpha(\mathbf{x}) = \iint_\Gamma \frac{\partial G(\mathbf{x},\mathbf{x}_l)}{\partial n} d\Gamma$ is the solid angle from which the closed surface Γ is seen from point $\mathbf{x} \in \partial\Gamma$ and $G(\mathbf{x}, \mathbf{x}_l)$ is a Green function.