Applied Ocean Research 63 (2017) 276–287

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

Numerical study on cnoidal wave run-up around a vertical circular cylinder

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ARTICLE INFO

Article history:
Received 30 September 2016
Received in revised form 7 December 2016
Accepted 4 January 2017

Keywords:
Cnoidal wave
Wave run-up
Boussinesq equation
Finite element method
Curved wall boundary

ABSTRACT

A finite element model of Boussinesq-type equations was set up, and a direct numerical method is proposed so that the full reflection boundary condition is exactly satisfied at a curved wall surface. The accuracy of the model was verified in tests. The present model was used to further examine cnoidal wave propagation and run-up around the cylinder. The results showed that the Ursell number is a nonlinear parameter that indicates the normalized profile of cnoidal waves and has a significant effect on the wave run-up. Cnoidal waves with the same Ursell number have the same normalized profile, but a difference in the relative wave height can still cause differences in the wave run-up between these waves. The maximum dimensionless run-up was predicted under various conditions. Cnoidal waves hold entirely distinct properties from Stokes waves under the influence of the water depth, and the nonlinearity of cnoidal waves enhances rather than weakens with increasing wavelength. Thus, the variations in the maximum run-up with the wavelength for cnoidal waves are completely different from those for Stokes waves, and there are even significant differences in the variation between different cnoidal waves.

1. Introduction

Because of the important role of vertical cylinders in coast and offshore engineering, wave diffraction around a vertical cylinder is an interesting problem that has long been investigated by researchers. A three-dimensional analytical solution for the problem was initially derived by MacCamy and Fuchs [1] on the basis of linear theory. However, many natural phenomena are difficult to predict with linear diffraction theory, which has prompted researchers to seek nonlinear methods. Nonlinear waves have entirely distinct properties at different water depths; thus, different wave theories are applied to different ranges of the water depth.

For waves at a deep or finite water depth, the Stokes theory is suitable. The diffraction of Stokes waves around a cylinder has been further investigated by many groups. One approach has been the analytical method. Researchers such as Raman et al. [2], Kim and Yue [3], and Kriebel [4] have proposed second-order diffraction theories to predict wave run-up around a cylinder; others have emphasized calculating the second-order wave forces on the cylinder, such as Lighthill [5], Molin [6], Demirbilek and Gaston [7], and Taylor and Hung [8]. The other approach has been numerical simulation. Isaacson and Cheung [9] and Bai and Teng [10] developed time domain methods and set up numerical models to simulate wave diffraction. Wave run-up around the cylinder can naturally be obtained in a numerical simulation.

The properties of shallow-water waves can be described by the cnoidal or solidary wave theory. In contrast to Stokes waves, few analytical theories have been presented for the diffraction of cnoidal waves around a cylinder. Isaacson [11] proposed a first-order theory for this problem and then used it [12] to predict cnoidal wave run-up distributions around cylinders, which he compared with experimental data. The good agreement validated the theory. Isaacson also examined the effect of the Ursell number on wave run-up. The Ursell number is a parameter that indicates the normalized profile of cnoidal waves and is defined as $U_r = (H/d)/(kd)^2$, where H is the wave height, d is the water depth, and k is the wave number. The first-order approximation was not suitable for further investigation; thus, Isaacson did not examine whether cnoidal waves of the same normalized profile (i.e., the same Ursell number) have different dimensionless run-up distributions owing to differences in the relative wave height H/d. Later, for wave diffraction around a cylinder in shallow water, some researchers such as De Vos et al. [13] and Lykke Andersen et al. [14] improved empirical formulae on the basis of physical model studies, and other researchers used numer-

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http://dx.doi.org/10.1016/j.apor.2017.01.006
0141-1187/

ical models to do studies. These numerical models are generally based on Boussinesq-type equations.

Boussinesq equations are the most effective mathematical tool for describing phenomena of shallow-water waves. The form of the equations applicable to varying topography was originally derived by Peregrine [15]. Many researchers subsequently improved the equations in order to make them applicable to deeper water. Representative works include those by Madsen and Sørenson [16], Nwogu [17], Beji and Nadaoka [18], Madsen and Schäffer [19], Agnon et al. [20], Madsen et al. [21–23], Chazel et al. [24], and Kim et al. [25]. In addition, Wei et al. [26] and Gobbi et al. [27] presented fully nonlinear Boussinesq equations. In general, Boussinesq-type equations comprise a mass conservation equation and two momentum equations in the x and y directions to handle the velocity components (u, v) in those directions. Problems are usually simplified from three dimensions to two dimensions with Boussinesq equations.

By using numerical models based on Boussinesq-type equations, researchers have simulated shallow-water wave diffraction around cylinders. However, the emphasis has been on the features of the resulting wave field. Wang et al. [28], Woo and Liu [29], Ning et al. [30], and Kazolea at al. [31] all simulated a wave field resulting from a solitary wave passing a cylinder and described its features. Sun et al. [32] and Liu et al. [33] used Boussinesq equation models to investigate the diffraction of irregular waves around cylinders. Jiang and Wang [34], Wang and Ren [35], Li et al. [36], and Zhao et al. [37] have investigated cnoidal waves. However, the former two focused on the features of the resulting wave field. Only the latter two presented a cnoidal wave run-up envelope around a cylinder, but the comparisons with laboratory data were not satisfactory. To our knowledge, no researchers have entirely examined the effect of wave nonlinearity on cnoidal wave run-up around a cylinder until now. This problem, which was not solved by Isaacson [12], has not yet been addressed. Therefore, it became the main objective of the present study.

One reason for the few investigations into cnoidal wave run-up around a cylinder may be that the treatment of the boundary condition on the cylinder surface is not adequate in Boussinesq equation models. Errors allowed when describing the resulting wave field may still cause problems when examining the wave run-up. In order to accurately calculate the wave run-up, the full reflection condition (i.e., normal velocity is zero) must be satisfied exactly on the cylinder surface. The above researchers took different measures to implement the full reflection condition at curved wall surfaces. The finite difference method is not flexible with regard to this problem. With a Cartesian grid, a stepwise approximation of the curved boundary is inevitable, which is certain to introduce large errors. An alternative method is to use an orthogonal grid, as done by Jiang and Wang [34] and Wang and Ren [35]. This can exactly represent the cylinder surface, but it is extremely difficult to map such an orthogonal grid for complicated geometry. In the finite volume method, the treatment of the boundary condition is rather complex at curved boundaries, and boundary fluxes must be modified at each time step, whether with a Cartesian grid as by Ning et al. [30] or with an unstructured grid as by Kazolea et al. [31]. The finite element method is also suitable for unstructured grids and easier to use to set up numerical models than the finite volume method. According to Engelman et al.'s [38] general technique, the finite element method seems to be more convenient for implementing the full reflection condition at curved wall surfaces in fluid problems through coordinate transformation. Certainly, a specific method for finite element models of Boussinesq-type equations still needs to be developed. Li et al. [36], Woo and Liu [29], Sun et al. [32], Liu et al. [33] and Zhao et al. [37] have all published specific methods. Except for the last group of researchers, who used a potential function to represent the boundary condition with a unique equation, all of these groups applied an iterative process between the x- and y- direction momentum equations to make the boundary condition approximately satisfied at curved wall surfaces. Zhao et al.'s method is markedly different from the others because their governing equations are in terms of the velocity potential instead of the velocity. However, a comparison of their predicted wave run-up distributions around a cylinder with Isaacson's [12] analytical solution indicated that these treatments are not satisfactory.

To deal with the problems of curved boundaries, we applied a finite element method to set up a numerical model based on Beji and Nadaoka's [18] improved Boussinesq-type equations and improved the handling of the full reflection condition so the boundary condition is satisfied fully rather than approximately at curved wall boundaries. Moreover, the present model also completely satisfies the additional full reflection condition (i.e., normal derivative of the wave elevation is zero). The good agreement between the cnoidal wave run-up distributions of the present prediction and Isaacson's analytical solution for different cylinders demonstrated that the improvement is successful.

Using this model, we examined the influence of wave nonlinearity on cnoidal wave run-up around a vertical circular cylinder. Isaacson [12] used first-order approximation to show that cnoidal waves with the same Ursell number hold the same wave nonlinearity with the same normalized profile. However, we further examined whether there is still a difference in the dimensionless wave run-up between these waves with the same Ursell number owing to differences in the relative wave height. In addition, dimensionless amplitudes of the maximum run-up were predicted under different conditions. The effects of the wave height and the wavelength on the maximum run-up of cnoidal waves on a cylinder were clearly demonstrated. Note that the variation in the maximum run-up with the wavelength is distinctly different from that for Stokes waves, and there is even a significant difference in the variation between cnoidal waves with different nonlinearity. This indicates that cnoidal waves have distinct properties from Stokes waves because of the strong influence of the water depth.

2. Numerical model

2.1. Governing equations

The present work was based on the improved Boussinesq equations derived by Beji and Nadaoda [18]. In the Cartesian coordinate system (x, y), the mass conservation equation is

$$\eta_t + \nabla \cdot [(d+\eta)\boldsymbol{u}] = 0, \tag{1}$$

and the momentum conservation equation in vector form is

$$\boldsymbol{u}_t + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + g\nabla\eta = \left(1+\beta\right)\frac{d}{2}\nabla\left[\nabla \cdot (d\boldsymbol{u}_t)\right] + \beta\frac{gd}{2}\nabla\left[\nabla \cdot (d\nabla\eta)\right] - \left(1+\beta\right)\frac{d^2}{6}\nabla\left(\nabla \cdot \boldsymbol{u}_t\right) - \beta\frac{gd^2}{6}\nabla\left(\nabla^2\eta\right), \tag{2}$$

where $\boldsymbol{u} = (u, v)$ is the two-dimensional depth-averaged velocity vector, $\eta(x, y)$ is the water surface elevation, $d(x, y)$ is the water depth, g is the acceleration of gravity, $\nabla = (\partial/\partial x, \partial/\partial y)$ is the two-dimensional gradient operator, β is a parameter for adjusting the accuracy of the linearized dispersive relation of the equations, and the subscript t denotes the time derivative. Li et al. [36] verified that the optimum linearized dispersive relation can be obtained at $\beta = 1/5$ up to the Padé [2, 2] approximant of the linear dispersion relation.

In order to avoid calculating the third-order spatial derivatives of the wave profile, we introduce two auxiliary variables: $a = \partial\eta/\partial x$