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Improved numerical solution of Dobrovol'skaya's boundary integral equations on similarity flow for uniform symmetrical entry of wedges

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ABSTRACT

Dobrovol'skaya [1] presented a similarity solution for the water entry of symmetrical wedges with constant velocity. The solution involves an integral equation that becomes increasingly harder to numerically solve as the deadrise angle decreases. Zhao and Faltinsen [2] were able to present reliable results for deadrise angles down to 4°. In this paper, Zhao and Faltinsen's results are improved and reliable results for deadrise angles down to 1° are confirmed by comparing to the asymptotic solutions at small deadrise angles and the solutions by the traditional boundary element method at relatively large deadrise angles. The present similarity solution results provide a reference solution in theoretical studies of water entry problems and in developing accurate numerical solvers for simulating strongly nonlinear wave–body interactions, which flows are governed by Laplace equation or Euler equation.

1. Introduction

Solid objects entering through a water (liquid) surface often involves large unsteady hydrodynamic loads and rapid deformation of free surface and is therefore of great interest to the design of ship bows, lifeboats, planning vessels, high-speed seaplanes, surface-piercing propellers and offshore or coastal structures. Wagner [3] studied water entry of wedges. He accounted for the local uprise of the water and presented details of the flow at the spray roots, which included predictions of maximum pressure. Wagner's first-order outer-domain solution does not include the details at the spray roots and overestimates the vertical hydrodynamic forces. For finite deadrise angles θ as defined in Fig. 1, Wagner used a flat-plate approximation, which leads to pressure singularities at the plate edges. Cointe and Armand [4] and Howison et al. [5] used matched asymptotic expansions to combine Wagner's inner-flow-domain solution at a spray root with an outer-flow-domain solution. In that way, the pressure singularities at the spray roots are removed. Cointe [6] studied also the jet domain and presented predictions of the angle between the free surface and the body surface at the intersection point for water entry of a wedge with constant entry velocity. The theoretical model by Cointe and Armand [4] or Howison et al. [5] only gives satisfactory solution for small deadrise angles. Faltinsen [7] studied water entry of wedges with larger deadrise angles. He accounted for the deadrise angle in constructing the outer domain solution, which results in significant improvement of the asymptotic solution for larger deadrise angles. The previously mentioned theoretical models give approximate analytical solutions for two-dimensional problems. By neglecting gravity, Dobrovol'skaya [1] presented the similarity solution, which exactly represents the water entry of symmetrical semi-infinite wedges with constant velocity within the framework of potential flow of incompressible liquid. Semenov and Iafrati [8] obtained similarity solutions for the water entry of asymmetric wedges without flow separation. For three-dimensional problems, Faltinsen and Zhao [9] presented asymptotic solutions for water entry of axisymmetric bodies; Scolan and Korobkin [10] presented exact analytical solutions to the Wagner problem; Wu and Sun [11] found the existence of similarity solutions in the case of an expanding paraboloid entering water.

Dobrovol'skaya's similarity solution is applicable for any deadrise angle. Its existence and uniqueness has been proved by Fraenkel and Keady [12]. The similarity solution has been widely used for a reference solution in theoretical studies of water entry problems and also in developing accurate numerical solvers for simulating strongly nonlinear wave–body interactions, for instance, by Mei et al. [13], Söding [14], Semenov and Iafrati [8], Wu [15] and Wang and Faltinsen [16]. When used as a reference solution, the similarity solution results should be accurate. The similarity solution is represented by a nonlinear singular integral equation, which is very difficult to solve. The challenges increase with reducing the deadrise angle, because smaller deadrise angles

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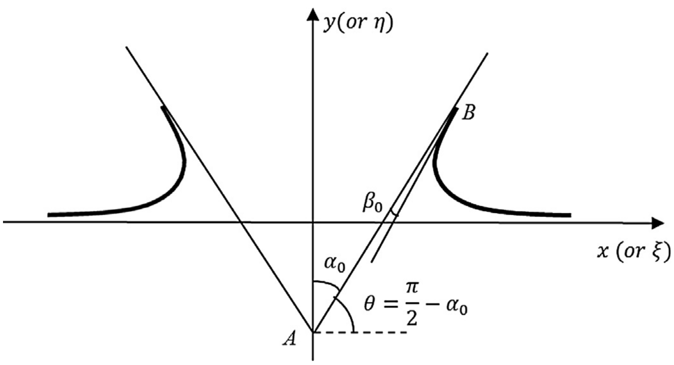

Fig. 1. Coordinate system and sketch of a wedge symmetrically entering into calm water. α_0 is half of the wedge angle; θ is the deadrise angle; β_0 is the angle between the body surface and the water surface at the intersection point B.

result in thinner and longer jet flows. Dobrovol'skaya [1] only presented results for deadrise angles equal to and larger than 30°. Zhao and Faltinsen [2] pointed out that Dobrovol'skaya [1]'s results for the deadrise angle of 30° are not accurate. They improved Dobrovol'skaya's results and obtained results in a wider range of deadrise angles (down to 4°). However, there was non-negligible discrepancy in the pressure distribution on the wedge surface when compared with the results by the boundary element method (see [2], Fig. 6). Due to numerical challenges, results for deadrise angles smaller than 4° have not been obtained yet. In this paper, a nested iterative method based on quasi-dynamic under-relaxation is proposed to derive accurate results of Dobrovol'skaya's similarity solution. By employing this method, we successfully obtained similarity solution results for deadrise angles down to 1°. The numerical error of the present similarity solution results has been estimated. The accuracy of the results is further confirmed by comparing to the asymptotic solutions at small deadrise angles and the solutions by the traditional boundary element method at relatively large deadrise angles. The present similarity solution results agree well with the asymptotic solutions at small deadrise angles and the discrepancy between the two solutions tends to vanish with decreasing the deadrise angle, which are expected when comparing well-developed asymptotic solutions to exact solutions. At relatively large deadrise angles, the present results coincide with those obtained by the traditional boundary element method, which improve Zhao and Faltinsen [2]'s results. The assumptions of the similarity solution must be kept in mind, such as a semi-infinite wedge is considered. It is noted that, at small deadrise angles, the airflow will cause the free surface to raise at the chines if the wedge is rigid with a finite length. The consequence is that air cavities are formed under the wedge bottom. However, there is more to it than that. Hydroelasticity will in practice matter [17]. Furthermore, liquid compressibility can matter for small deadrise angles. Anyway, it is important that numerical solvers are challenged in their testing phase. For this purpose, the accurate similarity solution results at small deadrise angles are good reference solutions to be used. They also provide good reference solutions for theoretical studies of water entry problems.

2. Mathematical model

2.1. Governing equation

To model the symmetrical entry of a semi-infinite wedge into the initially calm water, a Cartesian coordinate system is introduced: the x-axis is along the undisturbed water surface; the y-axis is along the body axis of symmetry and positive upwards. The coordinate system and sketch of the water entry problem are shown in Fig. 1.

The air flow is neglected. In case that the entry velocity is not high enough to make acoustic effects relevant, it is appropriate to assume that the water is incompressible after a very early stage [18]. Because of the short duration of impact, viscous effects are negligible provided that the Reynolds number is large. Further, the flow is irrotational as there is no initial vorticity. Therefore, a velocity potential $\varphi(x, y, t)$ of incompressible liquid satisfying Laplace's equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0 \tag{1}$$

is introduced. The kinematic free-surface condition is that a water particle remains on the free surface. The dynamic free-surface condition is that the water pressure is equal to the constant atmospheric pressure (surface tension is neglected). On the body surface, the normal velocity of the water is equal to that of the wedge body.

2.2. Similarity solution

Dobrovol'skaya [1] has presented similarity solutions for the water entry of symmetrical wedges with constant velocity. In the similarity flow, the velocity potential has the form

$$\varphi(x, y, t)=v_0^2 t\Phi(\xi, \eta), \tag{2}$$

where v_0 is the velocity of the wedge, $\xi = x/v_0t$, $\eta = y/v_0t$ and $\Phi(\xi, \eta)$ is a time-independent harmonic function. The function $\Phi(\xi, \eta)$ has to satisfy the kinematic free-surface condtion, the dynamic free-surface condition and the body-surface condtion, which are expressed as

$$\frac{\partial\Phi}{\partial\eta}-\eta\prime(\xi)\frac{\partial\Phi}{\partial\xi}+\xi\eta\prime(\xi)-\eta(\xi)=0, \tag{3}$$

$$\Phi-\xi\frac{\partial\Phi}{\partial\xi}-\eta(\xi)\frac{\partial\Phi}{\partial\eta}+\frac{1}{2}\left(\frac{\partial\Phi}{\partial\xi}\right)^2+\frac{1}{2}\left(\frac{\partial\Phi}{\partial\eta}\right)^2=0, \tag{4}$$

$$\frac{\partial\Phi}{\partial\xi}\cos\alpha_0-\frac{\partial\Phi}{\partial\eta}\sin\alpha_0=\sin\alpha_0 \tag{5}$$

respectively. The flow under consideration is then represented as a boundary-value problem. By using Wagner's h-function [3], the boundary-value problem can be reduced and solved by finding the solution of the nonlinear singular integral equation

$$f(s)=\frac{1}{\pi}\frac{c_0^2}{c^2}\int_0^s\frac{(1-t)^{-1-\alpha}\exp\left[t\int_0^1\frac{f(\tau)}{\tau(\tau-t)}\mathrm{d}\tau\right]}{\int_t^1 T^{-\frac{3}{2}}(1-T)^{-\frac{1}{2}+\alpha}\exp\left[-T\int_0^1\frac{f(\tau)}{\tau(\tau-T)}\mathrm{d}\tau\right]\mathrm{d}T}\mathrm{d}t, \tag{6}$$

where $\alpha = \alpha_0/\pi$ (see Fig. 1) and

$$\frac{c_0^2}{c^2}=\frac{\int_{\frac{1}{2}}^1 r^{-\frac{3}{2}}(1-r)^{-\frac{1}{2}+\alpha}(2r-1)^{-\alpha}\exp\left\{-\int_0^1\frac{f(\tau)}{\tau[\tau(2-(1/r))-1]}\mathrm{d}\tau\right\}\mathrm{d}r}{\int_{\frac{1}{2}}^1(1-r)^{-1-\alpha}(2r-1)^{-1+\alpha}\exp\left\{\int_0^1\frac{f(\tau)}{\tau[\tau(2-(1/r))-1]}\mathrm{d}\tau\right\}\mathrm{d}r}. \tag{7}$$

Once the function $f(s)$ is determined, the hydrodynamic problem can be considered as solved. Dobrovol'skaya [1] presented the free-surface elevation in terms of $f(s)$:

$$\left.\begin{matrix}\xi(s)\\ \eta(s)\end{matrix}\right\}=\left\{\begin{matrix}\xi_B+\\ \eta_B-\end{matrix}\right\}c\int_s^1 t^{-\frac{3}{2}}(1-t)^{-\frac{1}{2}+\alpha}$$

$$\exp\left[-t\int_0^1\frac{f(\tau)}{\tau(\tau-t)}\mathrm{d}\tau\right]\left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\}[\pi f(t)]\mathrm{d}t, \tag{8}$$

where the constants c and η_B have the form