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Dynamic response of a poro-elastic soil to the action of long water waves: Determination of the maximum liquefaction depth as an eigenvalue problem

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ABSTRACT

In this work, a theoretical analysis of the dynamic response of a poro-elastic soil to the action of long water waves is conducted. For some combinations of the physical parameters of the soil and the water waves, the vertical stress tends towards zero at a certain unknown depth in the soil, as measured from the top of that medium. Under this condition, the liquefaction of the soil is imminent, at which time the excess pore pressure is essentially equal to the overburden soil pressure. Physical problems of this type have been widely studied in the specialized literature. However, most major studies have focused on solving the governing equations together with a liquefaction criterion. Here, the maximum momentary liquefaction depth induced by long water waves is considered as part of the problem, which is treated as an eigenvalue problem. To solve this problem, the governing equations are written in dimensionless form. The theoretical results show that for long waves, the horizontal displacements are smaller in magnitude than the vertical displacements, and when the wavelength or wave period increases, the maximum liquefaction also increases. Analytical solutions for the excess pore pressure and the horizontal and vertical displacements are obtained. The analytical results for the pore pressure are found to be very close to the analytical results reported in the specialized literature.

1. Introduction

In recent years, the problem of wave-induced seabed instability has attracted considerable attention from coastal, geotechnical and structural engineers. The main reason for this is that many coastal structures have been damaged as a result of the wave-induced seabed response rather than construction deficiencies [1–3].

In the dynamic interactions between water waves and soils, ocean waves can generate hydrodynamic pressures that cause the pore water pressure and the effective stresses within the seabed to change drastically. These mechanisms will produce an increase in the pore pressure and a decrease in the effective vertical stress. Such a situation can lead to local seabed instability, including the well-known phenomenon of liquefaction. Once liquefaction occurs, soil grains may be carried off by the ocean bottom current like a fluid or may move in concert with wave action. According to the specialized literature, the phenomenon of seabed instability is common, occurring in shallow flows, offshore water, and even deep water. It is well known that wave-induced seabed instability is a major cause of damage to or destruction of offshore structures; see, for example, [4,5]. Based on Biot's theory of poro-elasticity, many investigations of the wave-induced dynamic response of a porous seabed to wave loading have been conducted since the 1970s [6–8]. Among these, Yamamoto et al. [9] and Madsen [10] derived a analytical solution for an infinite isotropic, poro-elastic seabed by treating the pore water and seabed as a compressible and deformable medium. For short-crested wave loading, Hsu and Jeng [11] derived an analytical solution for an unsaturated isotropic seabed of finite thickness; similar studies for an infinite seabed can be found in Jeng and Seymour [12,13]. Based on an isotropic poro-elastic seabed, Zhang and Jie [14] determined that the depth to which the seabed is affected by the wave hydrodynamic pressure is of the same order of magnitude as the wavelength. Considering the combined effects of waves and currents, Zhang et al. [15] obtained an analytical solution for evaluating the pore pressure and effective stress in marine sediments. Experimentally, Chowdhury et al. [16] and Liu et al. [17] presented the results of a series of one-dimensional tests performed to gain a better understanding of the wave-induced pore pressures in the

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vertical direction. In addition, thanks to advances in computational capabilities, the governing equations obtained based on Biot's consolidation theory have also been solved numerically. These studies were conducted to elucidate the effects of the interactions between water waves and deformable seabeds in more complex and more realistic cases, such as foundations and pipelines [18,19]. A relevant aspect of any analysis of the interaction between water waves and the seabed is the selection of a suitable theory that correctly models the dynamic response of the soil. In this context, Ulker et al. [20] revised three formulations, namely, the fully dynamic, partly dynamic and quasi-static formulations, and analysed the different ranges of the physical parameters for which they are applicable.

Another relevant aspect of an analysis of the dynamic response of sea soils to water waves is the momentary liquefaction depth. In this context, many researchers have proposed various liquefaction criteria for identifying the maximum momentary liquefaction depth. Among these, Zen and Yamazaki [21] suggested a criterion for soil liquefaction under the influence of a two-dimensional progressive wave for a one-dimensional elastic analysis. They considered that wave-induced liquefaction in soil occurs when the submerged weight of the soil skeleton is less than the upward seepage force exerted on it. This criterion was later extended to a three-dimensional elastic analysis by Tsai [22], and this extended version is applicable to a short-crested wave system. Recently, other relevant 3D liquefaction criteria have been reported in the specialized literature; see, for example, Ye [23], who obtained a liquefaction criterion based on the Mohr-Coulomb friction principle, and Qi and Gao [24], who later proposed a new liquefaction criterion for analysing the effects of the saturation degree of a soil on its wave-induced momentary liquefaction.

The aforementioned theoretical works have enabled the research community to obtain important physical insight into the influences exerted by various soil and water wave parameters on the dynamic soil response, as characterized in terms of the pore pressure and soil deformations. In addition, these works have proposed various liquefaction criteria that allow the identification, to a first approximation, of the momentary liquefaction depth, which is obtained in a sequential manner; that is, the pore pressure is calculated first, and then the liquefaction depth is obtained from the liquefaction criterion. In the opinion of the authors, however, the momentary liquefaction depth of a sandy seabed is an inherent part of the problem and must be obtained simultaneously with the pore pressure and the soil deformations; therefore, the main objective of this work is to obtain the momentary liquefaction depth of a sandy seabed simultaneously with the pore pressure and the soil deformations.

In this work, the partial governing equations are presented in dimensionless form, and the characteristic scales for the pore pressure and displacements are obtained by conducting an order-of-magnitude analysis. This procedure allows to identify which parameters are small and that could be ignored or treated approximately. The characteristic scales allow us to analyse, to a first approximation, how the dynamic response of the soil differs for different physical properties of the soil and different water wave characteristics.

Among the physical variables, the finite depth at which the liquefaction may occurs is unknown and with the selection of appropriate boundary conditions, the liquefaction length should be determined as a part of the problem. To solve the above, in this work the vertical axis is normalized with the unknown characteristic liquefaction depth, which leads to that in dimensionless variables the liquefaction phenomenon occurs just when the above dimensionless vertical coordinate assumes the value of unity. On the other hand, under the assumption that for some combination of the properties of the soil and water waves and for some unknown liquefaction depth, the soil liquefaction is imminent, the pressure at the liquefaction depth is calculated from the liquefaction criterio of Zen and Yamazaki [21]. This pressure cannot be calculated directly, because it is a function of the unknown liquefaction depth. To solve this second problem a dimensionless excess pore pressure is defined in terms of the ratio of the excess pore pressure to the overburden of the soil, when this ratio is equal to the unity the liquefaction may occurs. The previous procedure allows to express an unknown physical control volume into a known dimensionless control volume. The dimensionless governing equations are function of an unknown small parameter that is formed with the product of the wave number and the maximum liquefaction depth; this parameter is defined as an eigenvalue that must satisfies the boundary conditions. The obtention of the value of this parameter represents among others the main objective of the present work. Therefore, the mathematical modelling performed to obtain the maximum liquefaction depth is treated as an eigenvalue problem.

In this work, the maximum liquefaction depth is obtained for the wave troughs, because under this condition, the dynamic pore pressure is negative and the dynamic vertical stress is tensile, meaning that momentary liquefaction is likely to occur, as noted by Ye and Jeng [25] and Jianhong et al. [26]. Under this condition, the seepage force lifts above the soil column, and the soil particles are no longer in contact Wang et al. [27].

The governing equations used here are the well-known partly dynamic $u-p$ approximation, which is an extension of the quasi-static approximation proposed by Biot [6]. Analytical solutions for the dimensionless excess pore pressure and the horizontal and vertical displacements are obtained. The pore pressure obtained using the present mathematical model is compared with the classical analytical solutions of Yamamoto et al. [9], Hsu and Jeng [11] and Mei and Foda [28] and they were in good agreement. In addition, the analytical solutions are compared with the numerical solution of the governing equations, and the results are again found to be in good agreement.

This article is organized as follows: Section 2 presents the governing equations, which model the dynamic response of the soil due to the pressure fluctuations of the water waves, expressed in terms of physical variables. The dimensionless version of these equations and an order-of-magnitude analysis are described in Section 3. The boundary value problems for the excess pore pressure and the horizontal and vertical displacements are addressed in Section 4. Section 5 presents the results and discusses the effects of the wave characteristics and the soil properties on the maximum liquefaction depth. Finally, the principal conclusions of this study are summarized in Section 6.

2. Formulation

Let us consider an incident linear long water wave of wavelength λ and amplitude A_I, propagating from right to left over a porous seabed of finite depth. The water depth h is assumed to be constant. In the selected Cartesian coordinate system, the positive direction of the x axis is to the right, with the origin at the junction between the porous seabed and the impermeable seabed. The z axis points outwards in the normal direction to the seabed. In the physical model, the porous medium and the impermeable soil are identified by brown and grey regions, respectively; see Fig. 1. Under the assumptions that the seabed soil is a deformable porous medium and that this soil is a mixture of three phases, namely, a solid phase that forms an interlocking skeletal frame, a liquid phase that occupies a majority of the pore space, and a gas phase that sometimes occupies a small portion of the pore space, the soil skeleton and the pore fluid (including both liquid and gas) can together be regarded as a compressible medium.