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Surface wave predictions in weakly nonlinear directional seas

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ABSTRACT

We have employed laboratory and numerical experiments in order to investigate propagation of waves in both long and short-crested wave fields in deep water. For long-crested waves with steepness, $\epsilon = k_c a_c = 0.1$ (a fairly extreme case), reliable prediction can be performed with the modified nonlinear Schrödinger equation up to about 40 characteristic wavelengths. For short-crested waves the accuracy of prediction is strongly reduced with increasing directional spread.

1. Introduction

Predictions are needed in many areas of science and technology. Applications range from the weather forecast to the financial market and the height of the incoming waves that hit you in the sea. If we should propose a rough categorization one could say that the prediction is stochastic if repeated cases with the same input give different output due to some inherent randomness in such models. If identical input produces the same output one may talk about deterministic prediction. Deterministic prediction reaches the final state from the initial state by applying a deterministic propagation operator.

The physics of ocean waves has been thoroughly studied for many years, and provides an excellent basis both for stochastic and deterministic prediction on various spatial and temporal scales. In particular, the surface is weakly nonlinear with the leading order contribution being a space/time Gaussian field. Analysis of surface wave problems has led to a large number of nonlinear wave equations, some of which are used as propagators in the deterministic predictions studied below.

Dynamic positioning of vessels during sensitive offshore operations such as float-over installation, equipment lifting, LNG loading connection, ROV operations and helicopter take-off and landing will benefit from a real-time prediction of the local wave conditions in an order of seconds to a few minutes ahead. Real time prediction could also warn against freak waves and could enable a helmsman to decide how to maneuver vessels during dangerous situations within a limited time horizon (e.g. [4–6]). Another important application is enhanced extraction of ocean wave energy, where the real-time dynamic control of floating wave energy converters can benefit from the possibility of real-time prediction of excitation force into the future [10].

The majority of existing commercial systems rely on linear wave theory for prediction of the encountering waves. Obtaining input data in the open ocean with the quality demanded by nonlinear wave propagation models is currently not an easy task. Nevertheless, it is possible to obtain high quality input data in a wave basin, for instance, with arrays of several wave probes [21]. Much work has been done to validate nonlinear wave propagation models with long-crested wave experiments in a laboratory. Shemer et al. [26] studied evolution of unidirectional nonlinear wave groups in a wave tank with the cubic Schrödinger (NLS) equation [13,44]. They observed in deep as well as in relatively shallow water experiments that the NLS equation is able to capture the overall features of nonlinear wave group evolution. Trulsen and Stansberg [40] applied both the NLS and the modified nonlinear Schrödinger (MNLS) equation [8,38] in order to investigate the spatial evolution of unidirectional bichromatic waves in a wave tank for deep water. Certain wave properties, which could not be adequately described by the NLS, were successfully captured by the MNLS. They suggested that the MNLS can be used to predict the evolution of long-crested waves at least up to the dimensionless fetch $\epsilon^2 k_c x = 3$ compared to $\epsilon^2 k_c x = 1$ for NLS and linear wave theory, where k_c is the characteristic wavenumber and x is the evolution distance. Trulsen [36,37] later extended the consideration to evolution of irregular waves. The overall conclusions were similar to [40], the main difference being reduced prediction horizon for the irregular wave experiments. Moreover, Trulsen [36] suggested a theoretical prediction range of the MNLS equation for various sea states.

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Unlike many other nonlinear wave propagation models, Schrödinger equations are computationally efficient, but at the cost of a constraint in the spectral bandwidth. This prompted Shemer et al. [27,28] to investigate evolution of an initially broad spectrum that formally violates the assumptions behind the derivation of (M)NLS. They compared experimental results with simulations based on the unidirectional Zakharov equation [44] accurate to the third-order (fourth-order) in wave steepness and the NLS (MNLS) equations. For a sufficiently narrow spectrum, the Schrödinger equation yields good agreement with the Zakharov equation. However, for relatively broader initial spectrum the accuracy degenerates compared with the Zakharov equation, which is free of bandwidth constraints. A recent comparison between experiments of broad-banded unidirectional waves and the Zakharov equation show satisfactory agreement [25] suggesting the Zakharov equation as an effective computational tool for prediction. Shemer and Dorfman [24] reported comparison of experiments of unidirectional wave groups with both spatial and temporal version of the MNLS equation. In addition to good agreement between experiments and simulations, they observed a gradual transformation of an initially symmetric spectrum into a strongly asymmetric one, this is in agreement with [9].

Until recently, relatively few validation efforts have been made on nonlinear directional wave fields. Some of these describe direct large-scale phase-resolved computations of nonlinear ocean wave fields [42,43]. However, simulation of typical nonlinear ocean wave fields with this method requires long computation time, limiting its applicability for short-time wave prediction. Direct comparisons of numerical simulations based on Higher-Order Spectral method to wave tank experiments can be found in [7].

In this paper, we shall consider the prediction of both long and short-crested wave experiments with linear and nonlinear equations. We use bichromatic and irregular waves from Marintek to validate long-crested wave models [37]. We also use experiments from MARIN to validate directional seas [21]. Among our selection of models, and based on an ensemble average of several realizations, the MNLS equation gives better prediction of the experiments for long fetch. For the irregular long-crested wave experiments with steepness $\epsilon = k_c a_c = 0.1$ we achieve reliable prediction over a fetch of about 40 characteristic wavelengths. For short-crested wave experiments we find less accurate prediction for the same fetch.

2. Prediction theory

In our setting, the physical problem consists of predicting the surface elevation at one or several points based on the observations of the surface elevations at several other points. In addition to location, the timing will vary. The following sections discuss some of the methods to be used.

2.1. Linear least squares prediction (LSQ)

We first consider the well-known calibration/validation setting where the model is calibrated using data both from the observation and prediction locations. In a second step another independent data set may be used for validating the model. Let $\mathbf{X}$ denote the input observation of dimension $O \times N$, where O is the number of observation spatial locations and N is the number of discrete times. The output Y denotes the prediction, in the simplest case, for only one prediction location, it has the dimension $1 \times N$. The expression for the best linear prediction, Y_p, takes the form

$$Y_p = \mathbf{qX}, \tag{1}$$

where the weight $\mathbf{q}$ is chosen so as to make the prediction error, $e = Y - \mathbf{qX}$, as small as possible. Here $\mathbf{q}$ is a vector of size $1 \times O$.

The calibration step consists of determining the weight $\mathbf{q}$, which may then be applied to Eq. (1) so as to obtain a reasonable prediction of Y, Y_p, for a new sets of observations. The numerical problem thus consists of solving Eq. (1) for $\mathbf{q}$. However, the resulting system of equations may be overdetermined, and the system may not always have a unique solution. The aim is thus to consider the error expression $e = Y - \mathbf{qX}$ and write the least squares problem

$$\|e\|^2 = (Y - \mathbf{qX})(Y - \mathbf{qX})'. \tag{2}$$

If $\mathbf{X}$ has full rank, one may consider solving the so-called normal equations that appear from minimizing the linear least squares problem in Eq. (2)

$$\mathbf{q} = Y\mathbf{X}'(\mathbf{XX}')^{-1}. \tag{3}$$

Even if $\mathbf{X}$ has full rank, the normal equations may be numerically ill-conditioned when the condition number of $\mathbf{X}$ becomes large. Numerical treatment, such as QR-factorization of $\mathbf{X}$ is then necessary to avoid numerical problems (see, e.g. [12,22]).

Consider $X(t)$ and $Y(t)$ are measurements from two locations, where Y is delayed in time relative to X by t_0. In this case, Y should be compensated for the forward shift in time compared to X, say $Y = \eta(t + t_0)$, where η is the surface displacement at the selected location. The optimal time-shift, t_0 is determined such that the prediction error is minimized.

2.2. Predictions based on covariance matrices (COV)

This method is a generalization of the LSQ method allowing both $\mathbf{X}$ and $\mathbf{Y}$ to be interdependent stochastic variables. Moreover, the covariance matrices entering the formula may be estimated in several different ways. Assume that $\mathbf{X}$ and $\mathbf{Y}$ are real or complex multivariate stochastic variables with dimensions $O \times N$ and $M \times N$ respectively, where M is the number of prediction locations. In order to keep things simple, we shall in the following assume that all variables have zero mean. For this method, the prediction $\mathbf{Y}_p$ also takes the form given in Eq. (1) where in this case $\mathbf{Y}_p$ is a vector, and $\mathbf{q}$ is an $M \times O$-matrix determined by minimizing the error covariance matrix with respect to the partial ordering of positive definite Hermitian matrices (see, [1], for this elegant technique).

Let $\mathbf{X}$ and $\mathbf{Y}$ have covariance matrices

$$\Sigma_{\mathbf{XY}} = E[\mathbf{XY}^H] = \Sigma_{\mathbf{YX}}^H, \tag{4}$$

and similarly for $\Sigma_{\mathbf{XX}}$ and $\Sigma_{\mathbf{YY}}$. Superscript H means the Hermitian transposed. Recalling $\mathbf{e} = \mathbf{Y} - \mathbf{Y}_p$, the unique minimum for $\Sigma_{\mathbf{ee}}$ is obtained with

$$\mathbf{q} = \Sigma_{\mathbf{YX}}\Sigma_{\mathbf{XX}}^{-1} = \Sigma_{\mathbf{XY}}^H\Sigma_{\mathbf{XX}}^{-1}, \tag{5}$$

when $\Sigma_{\mathbf{XX}}$ is nonsingular ([1], Thm. 8.2.1). The optimal prediction may thus be written as

$$\mathbf{Y}_p = \Sigma_{\mathbf{YX}}\Sigma_{\mathbf{XX}}^{-1}\mathbf{X}, \tag{6}$$

and the minimal error covariance matrix becomes

$$\Sigma_{\mathbf{ee}} = \Sigma_{\mathbf{YY}} - \Sigma_{\mathbf{XY}}^H\Sigma_{\mathbf{XX}}^{-1}\Sigma_{\mathbf{XY}}, \tag{7}$$

where the last term on the RHS is the prediction covariance matrix, $\Sigma_{\mathbf{Y}_p\mathbf{Y}_p} = \Sigma_{\mathbf{XY}}^H\Sigma_{\mathbf{XX}}^{-1}\Sigma_{\mathbf{XY}}$. Thus,

$$\Sigma_{\mathbf{YY}} = \Sigma_{\mathbf{Y}_p\mathbf{Y}_p} + \Sigma_{\mathbf{ee}}, \tag{8}$$

[1]. In conclusion, we may therefore always decompose $\mathbf{Y}$ into the best linear predictor from $\mathbf{X}$, as shown in Eq. (6), and an orthogonal component, $\mathbf{e} = \mathbf{Y} - \mathbf{Y}_p$, which is completely uncorrelated. It is interesting to observe that $\mathbf{e}$ is additive and orthogonal to the prediction of $\mathbf{Y}$. Obviously, should $\mathbf{X}$ be orthogonal to $\mathbf{Y}$, the prediction is simply $\mathbf{Y}_P = 0$ and $\mathbf{e} = \mathbf{Y}$.