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Modelling of long waves generated by bottom-tilting wave maker

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ABSTRACT

In order to generate very long waves in laboratory, a bottom-tilting wave maker is designed and used at the University of Dundee. This new type of wave maker can produce waves longer than solitary waves in terms of the effective wavelength, which provides better long wave model. Nonlinear and dispersive numerical models are built for modelling the wave tank. A shock-capturing finite volume scheme with high-order reconstruction method is used to solve the governing equations. By comparing to the experimental measurements, the numerical models are verified and able to approximate the resulting waves in the wave tank.

1. Introduction

Over the last few decades, there have been great interests in tsunami behaviour in near shore region and coastal areas. Among these studies e.g., [10,26,17,15,6], solitary wave has been the most commonly used tsunami wave model theoretically and experimentally. Indeed one of the reasons that solitary waves have been so popular for such a long time was that they are relatively easy to generate in laboratory [9].

Solitary wave propagates in constant depth with permanent form, whose surface elevation is described as

$$\eta(x,t) = A_s sech^2[K_s(x-ct)], \quad K_s = \frac{1}{h_0}\sqrt{\frac{3A_s}{4h_0}}, \tag{1}$$

in the horizontal coordinate x and time t, where η, A_s, c and h_0 denote free surface elevation, wave height, phase velocity and static water depth, respectively. However, recent studies such as [20] show that wavelength-to-depth ratios of solitary waves are much smaller than that of tsunamis in reality in the respect of the effective wave period T_s and the effective wavelength L_s of solitary waves:

$$T_s = \frac{2\pi}{K_s c}, \quad \text{and} \quad L_s = \frac{2\pi}{K_s}. \tag{2}$$

In other words, the link between its effective wavenumber K_s and wave height A_s is not realistic. In particular, when tsunamis are approaching the beach, nonlinearity increases significantly, leading to skewness of waves, which is already beyond the KdV scale.

Piston-type wave makers are popular and widely used to generate long waves in laboratory, but the disadvantage is that the wavelength of the generated waves is limited by the stroke length L_p as shown in Fig. 1. Very few studies mention using bottom-wave-generator to simulate tsunami generation or create long waves. The one designed by [10] is well known, which was designed for generating solitary waves excited by positive bed motion under the control of a hydraulic servo-system. We note here that waves generated by sudden bottom motion have been studied in the context of impulsive sloshing in a partially-filled tank e.g., [14,16,28].

In the present study, a bottom-tilting wave maker at the University of Dundee is investigated. The wave maker is able to generate very long waves, considerably longer than the effective wavelength of solitary waves with same amplitude. The bottom-tilting wave maker is designed based on a simple idea that moving the entire bottom can generate waves as long as the tank itself, which should be the longest wave in any given tank. A schematic drawing of concept of the bottom-tilting wave maker is depicted in Fig. 1. Note that, in comparison with typical piston-type wave maker, the bottom tilting wave maker has much longer moving length L, which can produce longer waves. In addition, the generated waves have very short distance to arrive at the shoreline by using the adjustable slope, which can be used to model long wave run-up.

There are a variety of wave theories which have been adopted to examine long waves. In early stage, people evaluated the wave motion from the linear wave theory in tsunami studies. For instance, [12,13,27] and [26] have proposed the linear wave theory to approximate near and far field waves. Although, the linear wave theory is limited to situations where nonlinear effects are small for both near and far field waves, it is used as first approximation of long waves in this work.

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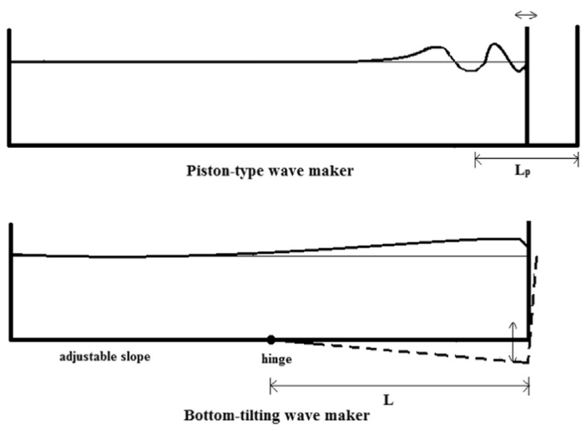

Fig. 1. Comparison between piston-type wave maker and the bottom-tilting wave maker.

When nonlinearity makes significant influence, the classical nonlinear shallow water (NSW) equations have been usually employed for simulating waves:

$$\left.\begin{array}{l}\eta_t + ((h+\eta)u)_x + h_t = 0,\\ u_t + uu_x + g\eta_x = 0,\end{array}\right\} \tag{3}$$

where η, u, h and g denote the free surface elevation, the depth-averaged fluid velocity, the static water depth and the gravitational acceleration, respectively. [5] proposed an analytical solution to NSW equations for monochromatic waves running up a beach with constant slope. An analytical solution was obtained by [26] for run-up of non-breaking solitary waves. Further development of the analytical solution has been made subsequently, e.g. [1,3,2,21].

On the other hand, frequency dispersion is of great importance during wave generation and propagation when pressure cannot be assumed hydrostatic. Many studies have shown that dispersive models have good performances on long wave simulation e.g., [23,30,8,7]. Hence, Boussinesq equations become a good choice to demonstrate the evolution of the surface waves, meanwhile both dispersion and nonlinearity are considered on the basis that they are both small and of the same order of magnitude. [8] introduced a variety of Boussinesq-type wave systems, among which some are applicable for flat bottom and some for arbitrary bottom. In this work, time-dependent bathymetry variations have to be coupled with surface wave. Therefore, the Boussinesq system derived by [29] for dynamic bathymetry is employed:

$$\left.\begin{array}{l}\eta_t + ((h+\eta)u)_x + h_t = 0,\\ u_t + g\eta_x + uu_x = \frac{1}{2}h(h_t + (hu)_x)_{xt} - \frac{1}{6}h^2 u_{xxt},\end{array}\right\} \tag{4}$$

which is an extension of the classical Boussinesq systems.

A wide range of numerical methods are developed in solving these hyperbolic equations, such as finite difference methods, finite element methods, finite volume methods and discontinuous Galerkin methods. [7] used a finite volume scheme to solve the Boussinesq equations for modelling surface waves due to underwater landslides. They demonstrated that finite volume method is good at approximating solutions to conservative equations with high efficiency, accuracy and robustness owing to its conservative and shock-capturing properties. Their numerical results have a good agreement with not only solitary wave propagation and interaction theoretically but also some experimental measurements. Since how to deal with the discontinuity of discrete solution at the cell interfaces is of key importance, [8] introduced three types of numerical fluxes which can take effect along with some reconstruction techniques such as TVD [25], UNO [11] and WENO [18] schemes. Among the three types, the central flux, as a Lax-Friedrichs type flux, is chosen in this work. Characteristic flux function is the one [7] used, and confirmed by [8] that it works as well as the central flux. [15] used Lax-Friedrichs flux splitting, which also shows a good performance. For reconstruction techniques, either UNO2 scheme [11] or WENO scheme uses adaptive stencil to interpolate the numerical flux and keep the piecewise polynomial representations always non-oscillatory. Note that, UNO2 scheme is of second order accuracy while WENO scheme can obtain higher-order accuracy.

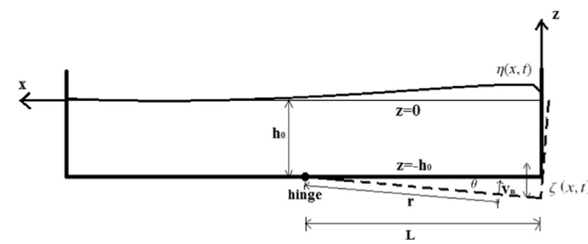

Fig. 2. Sketch of the two-dimensional wave maker.

The present study introduces the numerical modelling for this new bottom-tilting wave tank system. The fluid is under the assumptions of being inviscid, incompressible and irrotational flow. In addition, bottom dissipation is ignored and full reflection happens at the tank ends. In Section 2, linear wave theory is used for preliminary estimation. Then, in Section 3 the numerical models considering nonlinearity and dispersion for this specific wave tank are described in detail, including the numerical schemes and methods. By comparing the theoretical and experimental results, this numerical model is validated in Section 4. Finally, conclusion remarks are given in Section 5.

2. Preliminary estimation by linear wave theory

Fig. 2 shows the schematic sketch of the two-dimensional wave tank with a bottom moving in a combined rotating and lifting manner. Clearly, the analysis is divided into two parts at the toe of the slope (hinge). The moving bottom part will generate long waves, and the other part is for the generated waves propagating in the constant water depth or running up the slope. In this section, slope is not considered. The coordinate system origins at the end wall of the generation part, meanwhile the positive x axis is pointing the other end wall and z axis is pointing upwards. Thus, the fluid domain is bounded by the two end walls, the free surface and the bottom solid boundary, while the latter two are defined as $z = \eta(x, t)$ on the surface and $z = -h(x, t)$ at the bottom. For $0 < x < L$, water depth is expressed by $h(x, t) = h_0 + \zeta(x, t)$, where ζ denotes the bottom motion displacement, and h_0 the initial water depth.

Linear wave theory is useful for quick estimate of the generated waves, although limited by the conditions that it only applies to non-breaking waves and where nonlinear effects are small. The fluid motion in the wave tank can be described by the two-dimensional Laplace equation along with the simplified boundary conditions by linear wave theory. With Φ denoting the velocity potential, continuity equation reads

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0, \tag{5}$$

with boundary conditions introduced from the linearised kinematic and dynamic boundary conditions:

$$\frac{\partial\eta}{\partial t} = \frac{\partial\Phi}{\partial z}, \quad z = 0, \tag{6}$$

$$\frac{\partial\Phi}{\partial t} + g\eta = 0, \quad z = 0, \tag{7}$$

$$\frac{\partial\Phi}{\partial z} = 0, \quad z \cong -h_0. \tag{8}$$

Under the assumption that the moving bed is flat, solid and imperme-