Coastal Engineering 122 (2017) 124–140

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

Modeling transient long waves propagating through a heterogeneous coastal forest of arbitrary shape

Che-Wei Chang[a,*], Philip L.-F. Liu[a,b,c], Chiang C. Mei[d], Maria Maza[e]

[a] School of Civil and Environmental Engineering, Cornell University, USA
[b] Department of Civil and Environmental Engineering, National University of Singapore, Singapore
[c] Institute of Oceanic and Hydrological Sciences, National Central University, Taiwan
[d] Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, USA
[e] Instituto de Hidraulica Ambiental, Universidad de Cantabria, Spain

ABSTRACT

A model is proposed to study transient long waves propagating through coastal vegetation. The coastal forest is modeled by an array of rigid and vertically surface-piercing cylinders. The homogenization method, i.e. multi-scale perturbation theory, is applied to separate two contrasting physical length scales: the scale characterizing transient waves and the scale representing the diameter of and the spacing among cylinders. Fourier transform is employed so that the free surface elevation and velocity field are solved in the frequency domain. For each harmonic, the flow motion within a unit cell, consisting of one or more cylinders, is obtained by solving the micro-scale boundary-value problem, which is driven by the macro-scale (wavelength scale) pressure gradients. The cell-averaged equations governing the macro-scale wave amplitude spectrum are derived with the consideration of the effects of the cell problem solution. Similar to [1], the macro-scale wave amplitude spectrum is solved numerically with the boundary integral equation method, where a vegetated area is composed of multiple patches of arbitrary shape. Each forest patch can be divided into subzones according to different properties, such as planting pattern and vegetation size. Each subzone is considered as a homogeneous forest region with a constant bulk eddy viscosity determined by the empirical formula suggested in [13]. Once the solutions for wave amplitude spectrum are obtained, the free surface elevation can then be computed from the inverse Fourier transform. A computing program is developed based on the present numerical model. To check the present approach, we investigate several different forest configurations. However, we focus on incident waves with a soliton-like shape. We first re-examine the forest belt case. The numerical model is then checked by available theoretical results along with experimental measurements for two special forest configurations. For a single circular forest, the numerical results compare almost perfectly with the analytical solutions. The comparison with experimental data also shows very good agreements. The effects of different wave parameters on damping rate are discussed. The numerical model is further compared with the experiments for a forest region consisting of multiple circular patches. Good agreements are also observed between the simulated free surface elevations and the experimental measurements. The effectiveness of these two forest configurations on wave attenuation is discussed.

1. Introduction

Coastal vegetation has been considered as an effective means of shore protection for both short waves and long waves. To estimate wave attenuation by vegetation, laboratory observations as well as mathematical modeling on waves through coastal forests have been widely studied in recent years. Most of the existing literatures have been reviewed in [13,14]. However, in the interest of understanding the capability of coastal vegetation (e.g., mangroves) to dissipate long waves (e.g., tsunami waves, storm surges), several recent studies are worthy of mentioning here. For example, [3] conducted laboratory experiments on solitary waves through an array of emergent rigid cylinders. Different cylinder arrangements and sizes of array were tested. By measuring the free surface elevation, the solitary wave evolution across the target cylinder array was reported and the wave attenuation was shown. In addition, [5]'s experiments, using both artificial trees and cylindrical timber sticks, demonstrated the effectiveness of a coastal forest on reducing the maximum run-up of

* Corresponding author.
E-mail address: cc2338@cornell.edu (C.-W. Chang).

http://dx.doi.org/10.1016/j.coastaleng.2017.01.010
Received 7 August 2016; Received in revised form 14 January 2017; Accepted 14 January 2017

tsunami waves. [6] also examined the influences of vegetation density on wave dissipation and tsunami run-up in their experiments. Different parameterized models of mangroves were adopted in tsunami damping tests as well (e.g., [4] and [16]). Numerical approaches on interactions between solitary waves and vegetation have also been developed. For example, [10] proposed a three dimensional numerical scheme, directly simulating the interaction of solitary waves with emergent rigid cylinders. The flow field around each cylinder was computed from the direct simulation; however, the required computational efforts were considerable.

As discussed in [8], applying homogenization (multi-scale perturbation) theory provides higher computational efficiency compared with direct numerical approach. Separating the wave motion into two sharply contrasting scales, the macro-scale wave dynamics is solved based on cell-averaged equations, allowing better understanding of the physical process. For periodic waves, the semi-analytical model has been established for predicting the wave attenuation by specific configurations of cylinder array, e.g. an infinitely long forest belt ([13,14]) and a circular forest ([8]). Furthermore, [1] extended the model to a forest region of arbitrary shape by employing the boundary integral equation method. For transient waves, on the other hand, the model developed in [13] is limited to incident soliton-like waves propagating through a forest belt.

In this paper, we follow [1] and apply the homogenization theory to a transient long wave propagating through a forest region, which is composed of multiple patches of arbitrary shape. Each forest patch has different numbers of subzones due to different properties (e.g. vegetation type/size, arrangement and porosity). A subzone can be surrounded by other subzones and/or the open water region, where a sketch of a general forest region has been given in [1]'s Fig. 1 (it is also shown as Fig. S1 in the Supplementary materials). With the assumption of homogeneous forest subzones, a dimensional bulk eddy viscosity is obtained by modifying the empirical formula in [13]. Fourier transform along with the boundary integral equation methods are applied such that the arbitrary shape of patches/subzones can be resolved and the macro-scale wave amplitude spectrum is solved numerically. The free surface elevation is then obtained by inversely transforming the wave amplitude spectrum solutions back into the physical domain.

To check the present approach, several forest configurations have been investigated. For simplicity only the incident waves of soliton-like shape are considered. The forest belt studied in [13] is first re-examined with new analytical solutions. A homogeneous circular forest, used in [8] and [1] for studying periodic waves, is then employed to check the present approach and numerical model. Negligible differences are observed between the analytical solutions and the numerical results for these cases. The comparison between model results and experimental data is also presented, showing very good agreement. The relationship between the damping rate and the nonlinearity of incident wave is discussed. For further investigation of the skill of the numerical model, the experiments on multiple circular forest patches in [9] and [11] are simulated and good agreements with the experimentally-observed free surface elevations are obtained. The comparison of the effectiveness of a circular forest and multiple smaller patches on damping the incident wave is presented.

2. Formulation

Defining $\vec{x} = (x_1, x_2)$ as the horizontal Cartesian coordinates and z as the vertical coordinate pointing upwards from the mean water level, the free surface elevation can be expressed as $z = \eta(\vec{x}, t)$. Focusing on small-amplitude long waves, propagating in a constant water depth h_0, the leading-order velocities are on the horizontal plane and are independent of z, i.e. $u_i(\vec{x}, t)$, $i = 1, 2$. Therefore, the linearized equations ([13]) can be written as

$$\frac{\partial \eta}{\partial t} + h_0 \frac{\partial u_i}{\partial x_i} = 0 \tag{2.1}$$

and

$$\frac{\partial u_i}{\partial t} = -g\frac{\partial \eta}{\partial x_i} + \nu_e\left(\frac{\partial^2 u_i}{\partial x_j \partial x_j}\right) \tag{2.2}$$

in which the bottom friction has been neglected and the eddy viscosity ν_e is assumed as a constant.

In this paper we shall use the Fourier transform method to solve the governing equations and boundary conditions. As an example, the Fourier transform of the free surface elevation η can be expressed as

$$\hat{\eta}(x_i, \omega) = \int_{-\infty}^{\infty} \eta(x_i, t)e^{i\omega t}dt \tag{2.3}$$

where $\hat{\eta}(x_i, \omega)$ is called wave amplitude spectrum. The inverse Fourier transform of the wave amplitude spectrum gives back the expression of the wave surface profile:

$$\eta(x_i, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{\eta}(x_i, \omega)e^{-i\omega t}d\omega. \tag{2.4}$$

Here we introduce the dimensionless variables as

$$x_i^* = \frac{x_i}{\ell}, \quad t^* = \omega t,$$
$$\eta^* = \frac{\eta}{H_{\text{inc}}} \quad \text{and} \quad u_i^* = \frac{u_i}{\sqrt{g(h_0 + H_{\text{inc}})}\,H_{\text{inc}}/(h_0 + H_{\text{inc}})} \tag{2.5}$$

where the horizontal coordinates are scaled by the characteristic tree spacing ℓ (micro-length) and the free surface elevation is scaled by the incident wave height H_{inc}. The wave speed of a solitary wave, i.e. $\sqrt{g(h_0 + H_{\text{inc}})}$, is assumed. We first apply Fourier transform to the linearized Eqs. (2.1) and (2.2), and then normalize the results with the proper scales as indicated in (2.5) to yield

$$-i\varepsilon\hat{\eta}^* + \frac{\partial \hat{u}_i^*}{\partial x_i^*} = 0, \quad i, j = 1, 2 \tag{2.6}$$

and

$$-i\varepsilon\hat{u}_i^* = -\frac{\partial \hat{\eta}^*}{\partial x_i^*} + \varepsilon\sigma\frac{\partial^2 \hat{u}_i^*}{\partial x_j^* \partial x_j^*}, \quad i, j = 1, 2 \tag{2.7}$$

where $\hat{u}_i^*$ is the dimensionless velocity spectrum and the small parameter ε is defined as

$$\varepsilon = \frac{\omega\ell}{\sqrt{g(h_0 + H_{\text{inc}})}} \equiv k\ell \ll O(1). \tag{2.8}$$

We remark here that since the linear long wave theory is used, the incident wave height must be reasonably small, i.e. $H_{\text{inc}}/h_0 \ll O(1)$ has been applied. The wave dispersion is also assumed to be negligible. Therefore, the size of the forest should be in the same order of magnitude of the characteristic wavelength, which has to be much greater than the tree spacing.

Following [13], we propose that each forest subzone has a constant bulk value of dimensional eddy viscosity ν_e, which can be determined by the following empirical formula:

$$\nu_e = 1.86(1 - n)^{2.06}U_0\ell \quad \text{with} \quad U_0 = \sqrt{g(h_0 + H_{\text{inc}})}\left(\frac{H_{\text{inc}}}{h_0 + H_{\text{inc}}}\right) \tag{2.9}$$

where U_0 represents the depth-averaged horizontal water particle velocity. The corresponding dimensionless eddy viscosity can then be written as

$$\sigma = \frac{\nu_e}{\omega\ell^2} = 1.86(1 - n)^{2.06}\frac{1}{k\ell}\left(\frac{H_{\text{inc}}}{h_0 + H_{\text{inc}}}\right), \quad k = \frac{\omega}{\sqrt{g(h_0 + H_{\text{inc}})}}. \tag{2.10}$$

The dimensionless eddy viscosity σ is a function of ω. It should be noted that a similar formula was used in [13], where $U_0 = \sqrt{gh_0}H_{\text{inc}}/2h_0$ was specified.