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Estimation of incident and reflected components in highly nonlinear regular waves

Thomas Lykke Andersen *, Mads Røge Eldrup, Peter Frigaard

Department of Civil Engineering, Aalborg University, Thomas Manns Vej 23, DK-9220 Aalborg Ø, Denmark

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ABSTRACT

Knowledge of the incident and reflected waves present in laboratory experiments is a key issue in order to correctly assess the behaviour of the tested structure. Usual applied reflection separation algorithms are based on linear wave theory. These linear methods might result in unreliable estimates of the incident and reflected waves in case the waves are nonlinear. In the present short paper a new nonlinear reflection separation algorithm optimized for regular waves is presented. The method separates the superharmonics into bound/free and incident/reflected components. The separation in bound and free components is possible because they travel with different celerity. The new method is an extension of the Lin and Huang (2004) method as they used linear dispersion so indirectly assumed the bound waves to be of 2nd order maximum. They did thus not take into account the amplitude dispersive effect of nonlinear waves (3rd and higher order). The present method uses nonlinear wave celerity in order to overcome this limitation. It is shown in the present paper that for highly nonlinear regular waves none of the existing state-of-the-art tools are reliable. The new method showed on the other hand a good match for all of the tested synthetically generated wave conditions including shallow and deep water and proved also to be robust to noise. Even though the new method is developed for horizontal sea bed it is demonstrated to provide reasonable results on numerical data for vertical asymmetric waves on mildly sloping foreshores.

1. Introduction

In model test experiments it is a key element to accurately determine the incident waves the models are exposed to. For reflective structures this involves the separation of the incident and reflected wave trains. Alternatively waves might be calibrated without the structure in place, but this requires additional testing time and there is anyway no guarantee for the incident waves to be identical with the structure in place. This requires namely a perfect active absorption system to avoid re-reflection at the paddle and it also requires no interaction of incident and reflected waves. Wave breaking is one example of a phenomena that is dependent on the combination of incident and reflected waves and might therefore be different with and without structure in-place. Therefore, accurate and robust methods to determine the incident and reflected waves with the structure in-place are important.

The first significant work on separation of incident and reflected waves was the linear two-gauge methods of Goda and Suzuki (1976). This method was extended to a more robust 3 gauge linear method by Mansard and Funke (1980) in order to minimize effects of noise and singularities. Mansard and Funke method was further extended to arbitrary number of wave gauges by Zelt and Skjelbreia (1992).

The nonlinear LASA V model proposed by Figueres et al. (2003) is a local time domain solution. LASA is based on fitting the parameters of the incident and reflected profiles in each local time window by using simulating annealing as fitting algorithm. In LASA V the wave profile used in each local time window is a 5th order Stokes profile for both incident and reflected waves. This means basically an assumption of no free energy to be present which does not necessarily reflect physical model test conditions. The present paper demonstrates that LASA V results are not reliable for highly nonlinear waves even though tests cases have no free energy. Moreover, the computational cost of LASA V is a major downside of this method in practical applications.

All of above methods are applicable for regular and irregular waves, but might have problems with nonlinearities. For regular waves an alternative method was proposed by Lin and Huang (2004) that in addition to separate in incident and reflected primary components also consider free and bound superharmonics. However, their method is shown in the present paper to provide accurate results only for mildly nonlinear waves because amplitude dispersion was ignored.

* Corresponding author.
E-mail addresses: tla@civil.aau.dk (T. Lykke Andersen), mre@civil.aau.dk (M.R. Eldrup), pf@civil.aau.dk (P. Frigaard).

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An extension of the Lin and Huang method is proposed in the present paper by inclusion of amplitude dispersion in their method. Moreover, a problematic region where bound and free components have identical celerity is identified and a correction is suggested that is applicable to present method as well as Lin and Huang method.

The separation in bound and free harmonics is very relevant when analysing model tests. This is because if bound superharmonics are not properly generated by the wavemaker then unwanted free waves will exist. Unwanted free incident waves will also exist in case the active absorption system of the wavemaker is not ideal. Moreover, when the incident waves reach a structure part of the incident energy will be absorbed and part of it will be reflected. This means the reflected wave cannot bind the same amount of energy as the incident one. As a consequence reflected harmonics will always be partly free and partly bound. If the reflection is small or medium they will mainly be free, whereas if reflection is large bound reflected components might be significant. With the present method the details of the waves can be assessed including quality of the wave generation and active absorption systems, see for example Lykke Andersen et al. (2016). This is very important in order to correctly analyse model tests with regular waves.

2. Lin-Huang method

The bound superharmonic components travel with the celerity of the regular wave, but the free components travel with their own celerity as given by the dispersion equation. This makes it possible to separate bound and free components as suggested by Lin and Huang (2004). This separation is quite unique for this method and completely different to linear methods where all energy is assumed free. The mathematical model of the surface for N^{th} order waves as function of time (t) and coordinate of gauge number m (x_{m}) measured positive in the incident wave direction is taken as:

$$\begin{aligned}\eta(x_m,t) = {} & a_I^{(1)}\cos\left(kx_m-\omega t+\varphi_I^{(1)}\right)\\ & + a_R^{(1)}\cos\left(kx_m+\omega t+\varphi_R^{(1)}\right)\\ & + \sum_{n=2}^{N} a_{I,B}^{(n)}\cos\left[n(kx_m-\omega t)+\varphi_{I,B}^{(n)}\right]\\ & + \sum_{n=2}^{N} a_{R,B}^{(n)}\cos\left[n(kx_m+\omega t)+\varphi_{R,B}^{(n)}\right]\\ & + \sum_{n=2}^{N} a_{I,F}^{(n)}\cos\left[k^{(n)}x_m-n\omega t+\varphi_{I,F}^{(n)}\right]\\ & + \sum_{n=2}^{N} a_{R,F}^{(n)}\cos\left[k^{(n)}x_m+n\omega t+\varphi_{R,F}^{(n)}\right]\\ & + e_m(t)\end{aligned} \tag{1}$$

where subscript I and R represent incident and reflected components, respectively. Subscript B and F denote bound and free components. a is the amplitude, k the wave number, $\omega = 2\pi/T$ is the cyclic frequency, T the period of the wave (primary component) and φ the phase. k denotes the wave number of the primary components and $k^{(n)}$ the wave number of the n^{th} order free component. These were determined by linear dispersion:

$$(n\omega)^2 = gk^{(n)}\tanh\left(k^{(n)}h\right) \tag{2}$$

where g is the gravity acceleration and h the water depth. Lin and Huang do not explicitly mention how the wave number of the primary component k is determined, but indirectly it appears that it must also be based on linear dispersion (i.e. Eq. (2) with $n = 1$). Another reason for this is that the same value of k is used for incident and reflected waves. The linear dispersion is only valid to 2nd order so they indirectly have assumed mildly nonlinear waves (2nd order theory valid).

3. New method: the Lykke Andersen method

The Lin-Huang method uses the linear dispersion equation, while higher order waves are in reality amplitude dispersive. The consequence of neglecting the amplitude dispersion is that for highly nonlinear waves the estimated distribution of energy among components will be somewhat disturbed from reality. This is especially the case when using few wave gauges, but as demonstrated in this paper also when using an overdetermined system.

The present method corrects that by applying a higher order wave length estimate. The wave numbers of the primary component (and bound components) thus depends additionally on the wave height (H) and will thus be different for incident and reflected waves. The wave period to be applied in the dispersion relation is usually known beforehand as the method applies to lab data only, but can also be estimated based on the signals for example by zero down crossing analysis or from the frequency of the first Fourier coefficient with significant energy. However, the incident and reflected wave height needed for the nonlinear dispersion relation is part of the solution and is thus unknown to begin with. Therefore, the following iterative procedure is used for the wave length estimation:

1. Calculate the wave number based on linear dispersion, i.e. for infinitesimally small wave height. Use this estimate as a starting guess for the incident and reflected wave numbers (k_{I}, k_{R}).
2. Calculate 1st to N^{th} order incident and reflected components using Eqs. (3)–(10). For bound components use latest estimated incident and reflected wave numbers (k_{I}, k_{R}) calculated in either step 1 for first iteration or step 3 for following iterations. Free components will normally be of much smaller height and linear assumption is assumed valid for those.
3. Calculate the incident and reflected wave heights based on incident and reflected surface elevations including bound superharmonic components. Calculate updated values of incident and reflected wave numbers including amplitude dispersion (k_{I}, k_{R}). The nonlinear dispersion applied is Stokes V order theory when it is valid (deep water). If Stokes V is not valid wave number is calculated by stream function theory using Fenton and Rienecker (1980) method. It is believed that the wave length estimate of Chang and Lin (1999) could also be applied without significant degradation of the proposed method.
4. Repeat step 2 to 3 until convergence is obtained for the incident and reflected wave numbers.

The applied mathematical model is thus:

$$\begin{aligned}\eta(x_m,t) = {} & a_I^{(1)}\cos\left(k_Ix_m-\omega t+\varphi_I^{(1)}\right)\\ & + a_R^{(1)}\cos\left(k_Rx_m+\omega t+\varphi_R^{(1)}\right)\\ & + \sum_{n=2}^{N} a_{I,B}^{(n)}\cos\left[n(k_Ix_m-\omega t)+\varphi_{I,B}^{(n)}\right]\\ & + \sum_{n=2}^{N} a_{R,B}^{(n)}\cos\left[n(k_Rx_m+\omega t)+\varphi_{R,B}^{(n)}\right]\\ & + \sum_{n=2}^{N} a_{I,F}^{(n)}\cos\left[k^{(n)}x_m-n\omega t+\varphi_{I,F}^{(n)}\right]\\ & + \sum_{n=2}^{N} a_{R,F}^{(n)}\cos\left[k^{(n)}x_m+n\omega t+\varphi_{R,F}^{(n)}\right]\\ & + e_m(t)\end{aligned} \tag{3}$$