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Optimal causal control of wave energy converters in stochastic waves – Accommodating nonlinear dynamic and loss models

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ABSTRACT

Recent research has shown that when constrained to causality, the optimal feedback controller for an ocean wave energy converter (WEC) subjected to stochastic waves can be solved as a non-standard Linear Quadratic-Gaussian (LQG) optimal control problem. In this paper, we present a relaxation to the modeling assumptions that must be made to apply this theory. Specifically, we propose a technique that uses the principle of Gaussian Closure to accommodate nonlinear WEC dynamics in the synthesis of the optimal feedback law. The technique is approximate, in the sense that it arrives at a computationally efficient control synthesis technique through a Gaussian approximation of the stationary stochastic response of the system. This approach allows for a wide range of nonlinear dynamical models to be considered, and also accommodates many complex loss mechanisms in the power transmission system. The technique is demonstrated through simulation examples pertaining to a flap-type WEC with a hydraulic power train.

1. Introduction

It has long been recognized that control theory can be used to optimize the power generated by wave energy converters [1–7]. The determination of the optimal controller for a WEC system is predicated on knowledge of its dynamic behavior, as well as a characterization of the sea state to which it is subjected. For WECs with linear dynamic models, control designs typically presume harmonic waves, and are designed according to the same network-theoretic impedance-matching principles used in the design and operation of antenna arrays and waveguides [3]. However, true sea states are stochastic, with power spectra that exhibit significant available energy over a nontrivial band of frequencies [8]. For such cases, controllers derived via impedance matching theory must impose a feedback law which is the Hermitian adjoint (i.e., complex-conjugate transpose) of the hydrodynamic impedance matrix for the WEC, at all frequencies. For this reason, it is sometimes called "complex conjugate control," as in [5]. Such controllers are always anticausal, and thus require some anticipatory technique in which present decisions are made with future wave information. This can be accomplished, for example, with the use of deployable wave elevation sensors, coupled with model-predictive control techniques [9].

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Alternatively, controllers for WECs can be optimized subject to the constraint of causality. It was recently shown in [10] that under the assumptions of linear dynamics, a stationary stochastic sea state, and unconstrained generator controllability, the optimal WEC control problem is a special case of the Linear Quadratic Gaussian (LQG) control problem, which has a well-known solution. In this paper, we present an extension of the LQG theory for optimal causal WEC control, which can be used to accommodate nonlinearities in the system dynamics, and also can be used to compensate for complex loss mechanisms in the power train of the WEC.

2. Mathematical modeling

In this section we develop the modeling assumptions for the WEC dynamics, wave oscillations, and transmission loss models. These assumptions will carry over to the next section, in which an optimal controller will be synthesized, directly from the models developed here. We present these assumptions at the most generic level for which it is still possible to apply all the theory to follow. As such, we do not make specific assumptions here about the type of WEC being used, or the type of power train. However, for simplicity, we will assume that the WEC has only one power-takeoff (PTO) device. Extensions to the theory for multiple PTOs, embedded within a single WEC system, follow analogously, but require the introduction of an added layer of algebraic complexity in the mathematical presentation.

2.1. WEC dynamic model

To begin, let $v(t)$ and $u(t)$ be the "potential" and "flow" variables associated with the PTO device, in which $u(t)$ is presumed to be a variable which may be controlled. For example, supposing the PTO is a direct-drive electric machine, $u(t)$ would be the current in the stator coils of the machine (which can be controlled directly, via power electronics) and $v(t)$ is the back-EMF (i.e., internal voltage) of the machine. Likewise, supposing the PTO is a continuously-controllable hydraulic ram, $u(t)$ would by the hydraulic force of the ram, and $v(t)$ would be its extension velocity. Irrespective of the technology used, we make the assumption here that $u(t)$ may be varied continuously.

We then assume that the WEC dynamics can be described to adequate precision by a nonlinear, finite-dimensional state space model, i.e.,

$$\dot{x}_c(t) = \mathcal{F}_c(x_c(t), f_r(t), f_w(t)) + B_c u(t) \tag{1}$$

$$v(t) = \mathcal{H}_c(x_c(t)) \tag{2}$$

where $x_c(t)$ is the WEC state vector, n_c is the associated state dimension, $f_r(t)$ is a vector of forces which capture the radiation damping, $f_w(t)$ is a vector of forces which capture the incident wave excitation, and $\mathcal{F}_c(x_c, f_r, f_w)$ and $\mathcal{H}_c(x_c)$ are differentiable functions.

We assume that $f_w(t)$ is related to the wave elevation $a(t)$ via a linear convolution; i.e.,

$$f_w(t) = \int_{-\infty}^{\infty} h_w(t-\tau)a(\tau)d\tau \tag{3}$$

where $h_w(\cdot)$ is the associated convolution kernel. Note that in the above, we account for the well-known non-causality of the mapping between a and f_w [11,12], by the fact that the integration domain above is over the entire real line $\tau \in (-\infty, \infty)$. We assume that h_w is square-integrable over this domain.

Regarding wave elevation $a(t)$ we presume it to be a stationary stochastic process with known power spectral density $S_a(\omega)$, where ω is in rad/s, and with the normalization convention that $S_a(\omega)$ is related to the mean-square value of $a(t)$ via

$$\mathcal{E}[a^2] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_a(\omega)d\omega \tag{4}$$

where $\mathcal{E}[\cdot]$ denotes the probabilistic expectation of the argument. For this assumption, it follows that the resultant power spectral density of f_w, denoted $S_w(\omega)$, is

$$S_w(\omega) = \hat{h}_w(j\omega)S_a(\omega)\hat{h}_w^T(-j\omega) \tag{5}$$

where $\hat{h}_w(s)$ is the Laplace transform of convolution kernel $h_w(t)$; i.e.,

$$\hat{h}_w(s) = \int_{-\infty}^{\infty} h_w(t)e^{-st}dt \tag{6}$$

and $j = \sqrt{-1}$. Note that in the construction of $S_w(\omega)$ as above, it is immaterial that $h_w(\cdot)$ is a non-causal convolution kernel.

Pausing for a moment, we consider the implications of the non-causality of $h_w(\cdot)$ on the stochastic dynamic model we have described. Note that the input to the dynamic model is actually $f_w(t)$, which has spectral density $S_w(\omega)$. The wave elevation $a(t)$, although its spectrum $S_a(\omega)$ is important for the purpose of deriving $S_w(\omega)$, does not enter directly into the dynamic model. Meanwhile, there is a causal relationship between $f_w(t)$ and $x_c(t)$. We conclude that in the treatment of

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