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Effects of second-order hydrodynamics on the efficiency of a wave energy array

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ABSTRACT

This paper considers power absorption by a square array of four heaving truncated cylinders subject to monochromatic incident waves, where the hydrodynamic calculations are correct to second-order. This idealised wave energy converter (WEC) array is of particular interest as it supports a near-trapped mode which offers the opportunity for strong hydrodynamic interactions between WECs. However, for typical WEC geometries this mode can only be excited at second-order. The approach taken is to seek to maximise the significance of the second-order effects, though even in a rather extreme case (a near-trapped mode in shallow water) the maximum additional power is approximately 30% of the linear power, while in all other cases the additional power is much smaller.

1. Introduction

A large body of work has been carried out analysing wave energy converters (WECs) using linear potential flow theory (see, for example, [1]). Fewer attempts have been made to use second-order potential flow methods to analyse wave energy devices. Mavrakos et al. [2] analysed a single heaving truncated cylinder WEC both with and without a toroidal structure surrounding it. Calculations of first and second-order loads were used to calculate absorbed power, though the semi-analytical matched eigenfunction expansion method used was only applicable to fixed structures at second-order (though fixed and moving structures at first-order). The effect of the second-order forces was found to be very small, though most significant for long waves in shallow water, where the second order incident potential is larger. Bellew and Stallard [3] considered second-order forces on a 2-device array of hemispheres in irregular waves and found the second-order forces to be small. In contrast Nader et al. [4] found that second-order effects could become significant in an Oscillating Water Column (OWC) WEC.

The interaction of second-order incident waves and fixed structures has been addressed by a number of authors. Different methods of calculating the second-order force on an isolated fixed cylinder were developed [5–7]. The problem of second-order wave diffraction by a square four-column array of bottom mounted cylinders was studied by Malenica et al. [8] using a semi-analytical method. For such arrays they reported significant modifications (increases) of the forces on the cylinders in the array when compared to linear theory. Furthermore, with access to the free surface motions they were able to report near-trapping of the second-order waves, manifested in the form of large free surface elevations, at second-order frequencies

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equal to the known linear near-trapping frequencies for this problem [9]. The influence of these near trapped modes when excited at second-order has been emphasised in subsequent studies [10,11].

Applications of second-order potential flow theory to the moving body problem have been less common. The sum frequency forces on a freely floating hemisphere in bichromatic waves were investigated by Kim and Yue [12]. They found that the second-order term due to first order body motions was significant, though in general the relative phases of the various second-order terms lead to the total second-order force being relatively modest. First- and second-order heave, pitch and surge motions of a freely floating truncated cylinder in monochromatic waves were investigated by Zaraphonitis and Papanikolaou [13]. They noted that the second-order responses in this case had multiple resonant peaks due to interaction between first-order terms. Further investigations of moving bodies at second order have been conducted [14–16].

This paper considers power absorption by an array of four generic heaving WECs in monochromatic waves, in a square arrangement which can support a near-trapped mode. Power absorption by the array due to both first- and second-order excitation is considered, where the second-order forces are determined using a full moving-body formulation. With low water depth and where the near-trapped mode is excited this represents a situation in which maximal second-order power absorption might be expected.

2. Theory

2.1. Incident waves

In a potential flow formulation the governing equation for the velocity potential Φ is the Laplace equation

$$\nabla^2\Phi = 0 \tag{1}$$

which applies throughout the fluid. Although this governing equation is linear the free surface boundary conditions are nonlinear so a perturbation analysis is introduced and the velocity potential (and free surface elevation) are expressed in terms of a power series in terms of the small parameter ϵ (where $\epsilon \ll 1$ is equal to the wave slope kA) such that:

$$\Phi = \epsilon\Phi^{(1)} + \epsilon^2\Phi^{(2)} + \dots \tag{2}$$

The first- and second-order velocity potentials for the incident waves are then written as

$$\epsilon^m\Phi_I^{(m)} = \mathrm{Re}\{\phi_I^{(m)}e^{-\mathrm{i}m\omega t}\} \tag{3}$$

where m is 1 or 2 and

$$\phi_I^{(1)} = \frac{-\mathrm{i}gA}{\omega}\frac{\cosh k(z+h)}{\cosh kh}e^{\mathrm{i}kx} \tag{4}$$

while

$$\phi_I^{(2)} = \frac{-3\mathrm{i}\omega A^2}{8}\frac{\cosh 2k(z+h)}{\sinh^4 kh}e^{2\mathrm{i}kx}. \tag{5}$$

This second-order velocity potential is associated with a free surface profile with half the wavelength of the linear wave, leading to a complete free surface profile with flatter troughs and steeper crests (note that the second-order wave travels at the same speed as the linear wave). In deep water ($kh \to \infty$) the second-order incident potential vanishes, though the double frequency contribution to the free surface does not.

For the perturbation approach to be valid the second order terms must be small relative to the first order terms – in deep water this is true provided kA is small, as been assumed previously. In shallow water the depth becomes important and second-order theory may be applied only up to the limit given by the Ursell parameter

$$\frac{A/h}{(kh)^2} \lesssim \frac{1}{3} \tag{6}$$

beyond which cnoidal wave theory is more appropriate [17].

The wave power transported by the linear incident wave per unit width of wave front is [1]

$$J = \frac{\rho g^2 D(kh)}{4\omega}A^2 \tag{7}$$

where

$$D(kh) = \left\{1 + \frac{2kh}{\sinh 2kh}\right\}\tanh kh. \tag{8}$$

is a depth function which is unity in deep water.