International Journal of Marine Energy 18 (2017) 109–113

Contents lists available at ScienceDirect

International Journal of Marine Energy

journal homepage: www.elsevier.com/locate/ijome

Design diagrams for wavelength discrepancy in tank testing with inconsistently scaled intermediate water depth

Donald R. Noble [a,b,*], Samuel Draycott [a], Thomas A.D. Davey [a], Tom Bruce [b]

[a] FloWave Ocean Energy Research Facility, The University of Edinburgh, Edinburgh EH9 3BF, United Kingdom
[b] School of Engineering, The University of Edinburgh, Edinburgh EH9 3FB, United Kingdom

ARTICLE INFO

Article history:
Received 9 March 2017
Revised 13 April 2017
Accepted 17 April 2017
Available online 20 April 2017

Keywords:
Tank testing
Water depth
Froude scaling
Marine renewable energy

ABSTRACT

The well-known dispersion relation links the length and period of a water wave with the depth in which it propagates. When model testing in tanks, the water depth should be consistently scaled to correctly replicate the waves. While this is done routinely by scaling foreshore bathymetry in coastal engineering physical model studies, and is not significant for deep water scenarios, this is not always considered when testing marine renewable energy devices, which are often in intermediate depth. Where water depth is not scaled consistently there will be resulting errors in wave parameters including wavelength, steepness, celerity, group velocity, and power. Design diagrams are presented to quantify and visualise these discrepancies over a typical range for testing offshore renewable energy devices. This design tool will facilitate experimental planning, quantification of uncertainties, and correlation of model test results with field data.

1. Introduction

When re-creating waves in a tank using Froude scaling laws, it is important to consider the depth ratio between the deployment site of interest and the tank, and to scale this correctly where possible. There will of course be instances where this is not possible, for example the deployment site water depth is not known, the tank has a fixed depth, or there are other constraints on the model size.

Whilst it is common knowledge that the wavelength of a water wave is a function of water depth, there has been little published regarding the incorrect scaled reproduction of wavelength resulting from inconsistent depth scaling. This issue was mentioned in [1], and expanded upon in [2], but the authors are not aware of other published discussions.

A number of authors, including [3,4], highlight the issue of scaling water depth when tank testing in the context of modelling mooring systems. These do not, however, highlight the consequences for wavelength error. Water depth scaling in relation to distorted hydraulic models is discussed in [5], noting that these models cannot be used for the study of water waves, as wavelength depends on water depth. For similitude in waves, the horizontal and vertical scales must be the same.

This technical note highlights the potential discrepancy in wavelength, group velocity, and power by restating the relevant aspects of small-amplitude wave theory. A method for calculating and visualising the errors resulting from incorrectly scaling the water depth when testing is then presented. This is followed by a brief discussion of implications for testing, focusing on marine renewable energy converters, which may be particularly sensitive to this issue.

* Corresponding author at: FloWave Ocean Energy Research Facility, The University of Edinburgh, Edinburgh EH9 3BF, United Kingdom.
E-mail address: D.Noble@ed.ac.uk (D.R. Noble).

http://dx.doi.org/10.1016/j.ijome.2017.04.001
2214-1669/

2. Background theory

When re-creating waves in a test tank, the Froude scaling law is used to match the ratio of inertial to gravitational forces that dominate this problem. The ratio of depth at the site of interest to the tank depth is important because gravity waves in water of finite depth can only be correctly re-created when the water depth is also scaled. The wavelength, celerity, and group velocity are all influenced by water depth, which in turn affect wave steepness and power. If the depth is not correctly scaled, this will lead to frequency dependent errors in these parameters, as discussed below. This situation may arise from constraints in the test facility, or from the deployment site depth not being known or considered when the model testing was being conducted.

It is well known that the properties of water waves are related by the dispersion relation, Eq. (1)

$$\omega^2 = gk\tanh(kh) \tag{1}$$

where ω is rotational frequency, k the wavenumber, g acceleration due to gravity, and h water depth. The dispersion relation can also be expressed in terms of period T, and wavelength L, using Eq. (2) to obtain Eq. (3).

$$\omega = 2\pi/T, \quad k = 2\pi/L \tag{2}$$

$$\Rightarrow L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi h}{L}\right) \tag{3}$$

Eq. (3) gives a unique relationship between the three quantities of wavelength, period, and depth. Therefore, if the depth is not scaled consistently, then wavelength will also be incorrect for a given Froude scaled period.

The wavelength at a site L_{site} can be calculated from wave period and depth using the dispersion relation Eq. (3), expressed here in terms of wavelength and period at the site.

$$L_{site} = \frac{gT_{site}^2}{2\pi}\tanh\left(\frac{2\pi h_{site}}{L_{site}}\right) \tag{4}$$

Using Froude scaling, where λ is the scale factor, these properties at tank scale should thus be Eq. (5) giving Eq. (6).

$$T_{tank} = T_{site}\sqrt{\lambda}, \quad h_{tank} = h_{site}\lambda, \quad L_{tank} = L_{site}\lambda \tag{5}$$

$$L_{tank} = \frac{gT_{site}^2\lambda}{2\pi}\tanh\left(\frac{2\pi h_{site}\lambda}{L_{tank}}\right) \tag{6}$$

However, if the tank depth is not correctly scaled, $h_{tank} \neq h_{site}\lambda$, the (incorrect) wavelength in the tank L_{tank*} will instead be given by Eq. (7), assuming the period is correctly Froude scaled.

$$L_{tank*} = \frac{gT_{site}^2\lambda}{2\pi}\tanh\left(\frac{2\pi h_{tank}}{L_{tank*}}\right) \tag{7}$$

The error in wavelength ε_L is taken as the ratio of wavelength actually generated in the tank L_{tank*} to the correctly scaled wavelength $L_{tank} = L_{site}\lambda$.

$$\varepsilon_L \equiv \frac{L_{tank*}}{L_{tank}} \tag{8}$$

The group velocity of a wave C_g is a more complex function of wavelength and water depth, given by Eq. (9).

$$C_g = \frac{1}{2}\sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi h}{L}\right)}\left[1 + \frac{4\pi h}{L\sinh\left(\frac{4\pi h}{L}\right)}\right] \tag{9}$$

The error in group velocity can be computed in the same manner, by calculating $C_{g,tank*}$ based on the wavelength actually generated in the tank, and $C_{g,tank}$ from the correctly scaled wavelength. The error is simply the ratio between these, Eq. (10)

$$\varepsilon_{Cg} \equiv \frac{C_{g,tank*}}{C_{g,tank}} \tag{10}$$

The speed, or celerity, of an individual wave is given by $C = L/T$, and steepness by $S = H/L$. Provided the period and height are correctly Froude scaled, the corresponding relative errors in celerity and steepness will thus equal the error in wavelength. As wave power $P = E_A C_g$, where E_A is wave energy per unit horizontal area [5], the relative error in power will equal that of group velocity. Any discrepancy in wave power is particularly important in tank testing wave energy converters, as wave power (kW/m) is scaled by $\lambda^{2.5}$, magnifying the projected full scale power discrepancy.