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Stationary phase and practical numerical evaluation of ship waves in shallow water *

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Abstract: A simple and highly-efficient method for numerically evaluating the waves created by a ship that travels at a constant speed in calm water, of large depth or of uniform depth, is given. The method, inspired by Kelvin's classical stationary-phase analysis, is suited for evaluating far-field as well as near-field waves. More generally, the method can be applied to a broad class of integrals with integrands that contain a rapidly oscillatory trigonometric function with a phase function whose first derivative (and possibly also higher derivatives) vanishes at one or several points, commonly called points of stationary phase, with the range of integration.

Key words: Ship waves, fourier integral, numerical evaluation, stationary phase

Introduction

The far-field and near-field waves created by a ship, of length L, that travels at a constant speed V along a straight path in calm water of uniform finite depth D are considered, as is illustrated in Fig.1. Far-field ship waves in uniform finite water depth have been widely considered in the literature, notably in Refs.[1-7], within the classical framework of linear potential flow theory, which is the only practical option for predicting far-field ship waves. Linear potential flow theory also provides a realistic basis to determine the near-field flow at a ship hull surface[8-12].

Within the framework of linear potential flow theory considered here, the flow at a ship hull surface can be formally decomposed into waves and non-oscillatory local flow[8,13]. The local flow component in this basic flow decomposition vanishes rapidly away from the ship and is negligible in the far field, indeed at relatively small distances from the ship. The waves in the decomposition into waves and a local flow is an essential, in fact dominant, flow component in the near field, and is the only significant flow component in the far field. Numerical evaluation of the wave component is then a critical element of the computation of both far-field ship waves and near-field (potential) flow around a ship hull and of the related wave drag, sinkage and trim. Indeed, numerical evaluation of this major flow component has been widely considered[8,9,14-16].

Far-field, as well as near-field, ship waves are defined by a Fourier integral associated with a linear superposition of elementary waves, explicitly given in Refs.[8,9] and in the next section. The integrand of this Fourier representation of ship waves oscillates very rapidly in the far field, and even in the near field for typical relatively small values of Froude number

$$Fr \equiv V / \sqrt{gL} \tag{1}$$

where V and L denote the speed and the length of the ship, as was already noted, and g is the acceleration of gravity. A practical numerical method–inspired by the method of stationary phase–for evaluating the Fourier integral that defines (far-field and near-field) ship waves is given in Ref.[15] for deep water.

This numerical method is extended here to the more general case of shallow water, and moreover is

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modified. The modification, based on an explicit use of the dispersion relation to determine the points of stationary phase, yields a method that is simpler, as well as more general, than numerical method given in Ref.[15].

Although ship waves are considered here, the method previously given in Ref.[15] for ship waves in deep water, and the related method given here for ship waves in uniform finite water depth, can be applied more generally to a broad class of integrals of the form

$$\int_a^b A(t)\mathrm{e}^{\mathrm{i}h\varphi(t)}\mathrm{d}t\,,\ \ 1 \ll h \tag{2}$$

Here, the amplitude function $A(t)$ is presumed to vary slowly with the range of integration $a \le t \le b$. Moreover, the derivative $\varphi'(t)$ of the phase function $\varphi(t)$ vanishes (or more generally all the derivatives $\varphi^{(m)}$ with $1 \le m \le M$ vanish) at one or several points, called points of stationary phase, within the integration range.

1. Integral representation of ship waves

Ship waves in calm water of uniform depth D are now considered. The non-dimensional water depth d is defined as

$$d \equiv \frac{Dg}{V^2} \tag{3}$$

Finite water depth effects are only significant if $d < d_\infty \approx 3$ as is well known, and the water depth is then effectively infinite if $d_\infty < d$.

The waves created by the ship are observed from an orthogonal frame of reference and related coordinates (X,Y,Z) attached to the ship. The Z-axis is vertical and points upward, and the undisturbed free surface is taken as the plane $Z = 0$. The X-axis is chosen along the path of the ship and points toward the ship bow as is shown in Fig.1. The non-dimensional coordinates

$$\boldsymbol{x} \equiv (x,y,z) \equiv (X,Y,Z)\frac{g}{V^2} \tag{4}$$

$$\boldsymbol{\xi} \equiv (\xi,\eta,\zeta) \equiv \frac{(X,Y,Z)}{L} \tag{5}$$

are used further on. Hereinafter, $\boldsymbol{x}$ denotes a flow field point located within the flow region outside the ship hull surface Σ, and $\boldsymbol{\xi}$ denotes a point of Σ, as is noted in Fig.1.

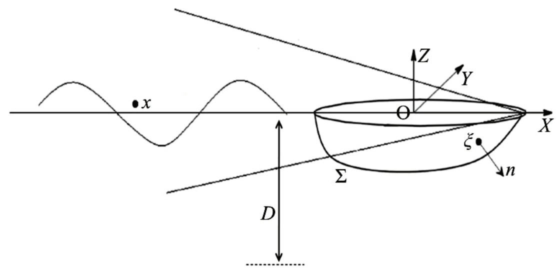

Fig.1 Ship waves created by a ship that travels at a constant speed V along a straight path in calm water of uniform finite water depth D

The Cartesian coordinates x and y in Eq.(4) can be expressed in the polar form

$$x = -h\cos\psi\,,\ \ y = h\sin\psi \tag{6}$$

where the horizontal distance $h \equiv Hg / V^2$ and the ray angle ψ are defined as

$$h \equiv \frac{Hg}{V^2} \equiv \sqrt{x^2 + y^2} \ \text{ and } \ \tan\psi \equiv \frac{y}{-x} \tag{7}$$

The ray $\psi = 0$ corresponds to the track $x < 0$, $y = 0$, $z = 0$ of the ship.

Within the framework of linear potential flow theory considered here, ship waves can be represented in terms of linear superpositions of elementary wave functions.

$$\varepsilon_\pm \equiv \frac{\cosh[k(z+d)]}{cosh(kd)}\mathrm{e}^{\mathrm{i}h\varphi_\pm} \ \text{ with}$$

$$\varphi_\pm \equiv \frac{\alpha x + \beta y}{h} \equiv \pm\beta\sin\psi - \alpha\cos\psi \tag{8}$$

and $(\alpha,\beta,k) \equiv (A,B,K)V^2 / g$. Moreover, the Fourier variables α and β and the wave number $k \equiv \sqrt{\alpha^2 + \beta^2}$ satisfy the dispersion relation.

$$\alpha^2 = kt\,,\ \ t \equiv \tanh(kd) \tag{9}$$

One has $t \approx 1$ in deep water $d_\infty < d$. The dispersion relation Eq.(8) and the identity $k \equiv \sqrt{\alpha^2 + \beta^2}$ show that the Fourier variables α and β in Eq.(10) are defined in terms of the wave number k via relations

$$\alpha = \sqrt{kt}\ \ ,\ \ \beta = \sqrt{k(k-t)} \tag{10}$$