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MPM simulations of dam-break floods*

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Abstract: Owing to its ability of modelling large deformations and the ease of dealing with moving boundary conditions, the material point method is gaining popularity in geotechnical engineering applications. In this paper, this promising Lagrangian method is applied to hydrodynamic problems to further explore its potential. The collapse of water columns with different initial aspect ratios is simulated by the material point method. In order to test the accuracy and stability of the material point method, simulations are first validated using experimental data and results of mature numerical models. Then, the model is used to ascertain the critical aspect ratio for the widely-used shallow water equations to give satisfactory approximation. From the comparisons between the simulations based on the material point method and the shallow water equations, the critical aspect ratio for the suitable use of the shallow water equations is found to be 1.

Key words: Material point method, dam-break floods, Lagrangian method, shallow water equations

Introduction

The material point method (MPM) is an extension of the fluid implicit particle (FLIP) method which was developed from the particle-in-cell (PIC) method to overcome many of its inherent numerical problems in the early implementations. By introducing the Lagrangian particles, the FLIP method eliminates the need to discretise the convective term which is a major source of numerical error in the PIC method[1]. Because of the Lagrangian material description, the FLIP can successfully track material discontinuities and model highly distorted flows[2-4]. The extension of the FLIP method to MPM was originally motivated by the problems of solid mechanics involving history-dependent materials. A distinct feature of the MPM method is that the governing equations are presented in the weak formulation, contrary to the FLIP method. This feature enables the MPM to be consistent with the finite element method (FEM) which has been the dominating computational method in geotechnical engineering. Another difference is that the constitutive equations are invoked at the material points in MPM, whereas in FLIP method, they are solved at the fixed grid nodes[3]. Recently, the MPM has been further extended to handle multi-phase problems with a single layer of material points[5-7] or two layers of material points[8-10], and they have been extensively applied to solve engineering problems such as progressive levee failure, pore water pressure development under wave attack on sea dike, seepage flow through embankment, underwater pipeline erosion, etc..

The MPM has many attractive advantages over traditional numerical methods. First, it is convenient to incorporate time-dependent constitutive models because information such as strain, stress, and other time-dependent variables can be carried by the moving material points, which enables the spatial and temporal tracking of the history of the material motion. Second, the use of a background mesh allows for the implementation of boundary conditions in a manner similar to the FEM, which is a big advantage compared with other mesh-free methods. In addition, the MPM is free of tensile instability that is evident in smoothed particle hydrodynamics (SPH)[11,12].

In hydrodynamics, free surface hydrodynamics is of significant industrial and environmental importance, but challenges arise in the implementation of surface

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boundary conditions on an arbitrarily moving water surface[13]. Due to the fact that the MPM combines the advantages of the mesh-based method and mesh-free method, it is an attractive method for computing free surface flows. However, despite the above mentioned advantages, there has been little application of the MPM in the field of hydraulic engineering.

Hence, the purpose of this study is to extend the MPM application to free water flow problems and explore the hydrodynamics of the dam-break flows. For the purpose of model validation, the MPM simulations are compared with experimental data and other computational results. Then, a parametric study of the idealised dam-break flow is conducted using the MPM. By comparing its predictions with those based on Shallow Water Equations (SWEs), it is found that the SWEs tend to overestimate the propagation speed of the dam-break flow for initial water columns of large aspect ratios. When aspect ratio of the initial water column is less than 1, then the SWEs give satisfactory predictions. Through these studies, the MPM proves to be a useful tool for investigating hydrodynamic problems.

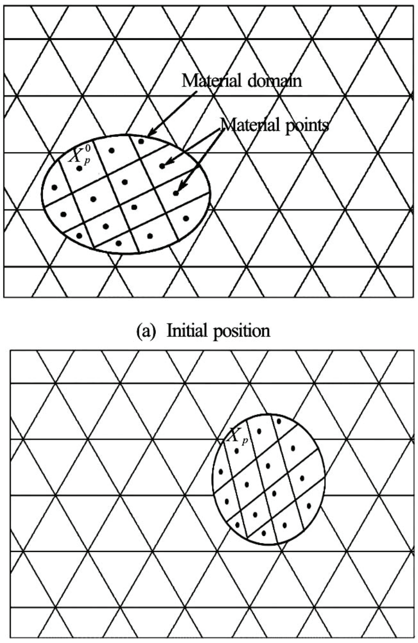

Fig.1 Sketch of background computational grid and the movement of material points

1. MPM model

1.1 *Governing equations*

Let the material domain Ω be represented by N_p number of material points (Fig.1) and let $\boldsymbol{x}_p$ denote the position of a material point in the current instant (i.e., time t). The same material point in the initial time $(t=0)$ is denoted as $\boldsymbol{x}_p^0$.

The governing equations which describe the motion of the continuum body Ω are the standard conservation equations: mass (Eq.(1)), and momentum (Eq.(2))

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}+\rho\nabla\cdot\boldsymbol{v}=0 \tag{1}$$

$$\rho\boldsymbol{a}=\nabla\cdot\boldsymbol{\sigma}+\rho\boldsymbol{b} \tag{2}$$

where $\rho(\boldsymbol{x},t)$ is the density of the continuum body, acceleration $\boldsymbol{a}(\boldsymbol{x},t)$ is the material derivative of the velocity, $\boldsymbol{v}(\boldsymbol{x},t)$ and $\boldsymbol{\sigma}$ is the Cauchy stress tensor that can be decomposed into pressure and deviatoric components. Water pressure is linked to its density change

$$p=p_0+K\frac{\rho}{\rho_0} \tag{3}$$

where K is the bulk modulus of water, p is pressure, and the symbols with a subscript indicate the initial reference values.

In the MPM, the non-linear convective term is not present in the formulation as a result of the overall Lagrangian framework[3]. On the contrary, the positions of the material points are updated each time step.

1.2 *Weak form of the governing equations*

Each material point carries the field variables, mass M_p, density $\rho_p(\boldsymbol{x},t)$, velocity $\boldsymbol{v}_p(\boldsymbol{x},t)$ and the Cauchy stress $\boldsymbol{\sigma}_p(\boldsymbol{x},t)$ $(p=1,2,3\cdots N_p)$. It is worth noting that the mass M_p is constant throughout the computation and therefore mass is conserved automatically in MPM. Density of the continuum body at the current instant can be expressed as

$$\rho(\boldsymbol{x},t)=\sum_{p=1}^{N_p}M_p\delta(\boldsymbol{x}-\boldsymbol{x}_p) \tag{4}$$

where $\delta()$ is the Dirac delta function. Then, we have the following relationship

$$\int\rho\mathrm{d}V=\sum M_p\delta(\boldsymbol{x}-\boldsymbol{x}_p)\mathrm{d}V \tag{5}$$

The weak form of the momentum equation can be obtained by multiplying the equation of conservation of momentum by a test function $\boldsymbol{w}$ as

$$\int_\Omega(\rho\boldsymbol{w}\cdot\boldsymbol{a}-\boldsymbol{w}\nabla\cdot\boldsymbol{\sigma}-\rho\boldsymbol{w}\cdot\boldsymbol{b})\mathrm{d}V=0 \tag{6}$$