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Poroelastic solid flow with double point material point method*

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Abstract: This paper presents the numerical modelling of one and two-dimensional poroelastic solid flows, using the material point method with double point formulation. The double point formulation offers the convenience of allowing for transitions in the flow conditions of the liquid, between free surface flow and groundwater flow. The numerical model is validated by comparing the solid flow velocity with the analytical solution. The influence of the Young's modulus on the solid flow velocity is discussed for both one and two-dimensional analysis cases. The effect of the shape of the two-dimensional solid is investigated. It is shown that the solid stiffness has an effect on the poroelastic flow velocity, due to swelling and bending for the one and two-dimensional cases, respectively. The shape is found to be an important factor on the flow velocity of the poroelastic solid.

Key words: Material point method, geocontainers, double point formulation, large deformations

Introduction

The modelling of large deformations is of great importance for engineering problems. The material point method (MPM) is a mesh-free method that has beendeveloped to address the problem of large deformation on a continuum level. The continuum material is represented by a set of Lagrangian points (material points) that move through an Eulerian background mesh. The material points contain all the properties of the continuum, such as mass, stress, strain and material parameters. Therefore, MPM can be seen as a combination of both Lagrangian and Eulerian formulations. The problems related to mesh distortion under large deformations are circumvented, as well as the diffusion associated with the convective terms of the Eulerian approach[1,2]. The use of Lagrangianmaterial points conserves mass and allows history-dependent material models to be used. The discrete equations for the momentum balance equations are obtained on the background grid similar to the finite element method with an updated Lagrangian formulation.

MPM has been used with success for geomechanics problems using the single point formulation[3-5], where both the solid and liquid are described by the same Lagrangian material point. However, this formulation is not appropriate to simulate problems involving the interaction between solid and liquid, as often is the case for offshore applications. To overcome this limitation, a double point formulation has previously been developed[6,7]. This formulation extends the classical two-phase approach to model saturated solids[8], which cannot capture the transition in state for both liquid and solid phases. The key aspect of this new formulation is that it considers two sets of Lagrangian material points to represent solidand liquid. The motion of both sets of material points is described by means of the momentum balance equations, using separate field velocities for both the solid and liquid phases. This means that each of the two constituents can move in respect to each other and both interact by means of a drag force.

Using the double point formulation, problems involving the fluidisation and sedimentation of solid-liquid mixtures can be modelled, as well as problems where free surface liquid flows through a porous solid (i.e., a transition ofthe liquid state from free surface flow to groundwater).

Geocontainers are sand filled bags encapsulated by a geotextile membrane, often used for coastal applications, such as revetments and breakwaters, mainly due to its resistance to erosion. The modelling of geocontainers is one of the many offshore applications

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that illustrates the necessity of the double point formulation. Although the single point formulation is not the most appropriate strategy to model the installation of geocontainers, it has been the most widely used methodology[9-12]. The need for the double point formulation is related to the requirementto model both large deformations, and the interaction between the solid and liquid materials, and the change in flow conditions of the liquid (between free surface and groundwater flow), as the liquid flows around and through the solid.

In the present paper, two problems where the transition of the liquid conditions occurs are analysed: one- and two-dimensional poroelastic flow through a porous solid immersed inside a Newtonian liquid. The latter resembles a geocontainer. The influence of the Young's modulus on the flow velocity of the falling solid by gravity, and the deformation of the modelled geocontainer will be discussed.

1. One-dimensional poroelastic solid flow

1.1 *Analysis description*

The numerical analyses of poroelastic solid flow have been performed with the MPM software Anura3D[13], following thedouble point formulation to simulate the interaction between solid and liquid[7]. The validation of the MPM implementation has been performed by simulating an one-dimensional poroelastic solid falling through a Newtonian liquid by gravity. This problem was chosen as it offers the convenience of having a closed form analytical solution for the steady state velocity of the poroelastic solid flow.

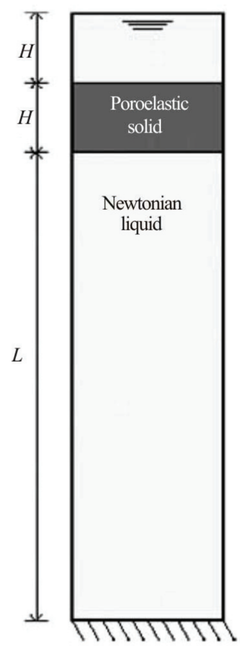

Fig.1 Geometry of the poroelastic solid and water column

The geometry of the problem is shown in Fig.1. The dimensions H and L were assumed as 0.1 m and 1 m, respectively. The system of equations is solved explicitly in the time domain. The domain was discretised using low-order tetrahedral elements (in total 780 elements), with initially ten solid and ten liquid material points for the elements filled with saturated poroelastic solid and ten liquid material points for the elements filled with liquid only. The nodes at the bottom of the column were fixed in the vertical direction.

The solid material is assumed to be linear elastic and the liquid material is modelled as compressible Newtonian liquid. Table 1 presents the material properties.

Table 1 Material properties for the poroelastic solid flow

Parameter	Value
Density solid, ρ_s / kg·m^{-3}	2 700
Solid Young modulus, E_s / kPa	10^4
Solid Poisson ratio, ν	0.3
Solid initial porosity, n_0	0.4
Solid grain size diameter, D_s / m	2×10^{-3}
Liquid density, ρ_l / kg·m^{-3}	1 000
Liquid bulk modulus, K_l / kPa	2×10^4
Liquid viscosity, μ_l / kPa·s	10^{-6}

1.2 *Analytical solution*

The steady state velocity of a rigid poroelastic solid falling through a liquid by gravity offers the convenience of an analytical solution[14]. The velocity $\bar{v}$ of the solid follows

$$\bar{v} = -\frac{\gamma'}{\dfrac{\mu}{\kappa} + \rho_l \dfrac{F}{\sqrt{\kappa}}|\bar{v}|} \tag{1}$$

where γ' represents the submerged volumetric weight, κ the intrinsic permeability, and F the coefficient from Ergun's drag force[15]

$$F = \frac{B}{\sqrt{A}n^{3/2}} \tag{2}$$

with parameters A and B being, respectively, 150 and 1.75[15]. The intrinsic permeability κ is defined as[16]

$$\kappa = \frac{D_s}{A}\frac{n^3}{(1-n)^2} \tag{3}$$