Available online at www.sciencedirect.com
ScienceDirect
Journal of Hydrodynamics
2017,29(4):621-631
DOI: 10.1016/S1001-6058(16)60775-8
www.sciencedirect.com/
science/journal/l0016058

Application of binding theory for seepage of viscoelastic fluid in a variable diameter capillary*

Er-long Yang (杨二龙)[1], Ting-ting Gu (谷婷婷)[2], Mei Wang (王梅)[3], Huan Li (李欢)[4]
1. *Key Laboratory on Enhanced Oil and Gas Recovery of the Ministry of Education, Northeast Petroleum University, Daqing 163318, China, E-mail: yel13796988396@126.com*
2. *Daqing Oilfield of China National Petroleum Corporation, Daqing 163000, China*
3. *School of Computer and Information Technology, Northeast Petroleum University, Daqing 163318, China*
4. *Second Oil Production Plant of Daqing Oilfield of CNPC, Daqing 163511, China*

(Received May 7 2015, Revised November 11, 2015)

Abstract: The polymer solution for polymer flooding is a viscoelastic fluid. There exist both shear flow and elongational flow when the polymer solution flows in a porous medium, where an additional dissipation is involved. The additional dissipation caused by elongational deformation is often ignored while studying the flow of the fluid in a porous medium. For a complex polymer solution, the generated elongational pressure drop cannot be ignored. In a capillary of fixed diameter, the polymer solution is only impacted by the shear force, and its rheological property is pseudoplastic. Therefore the variable diameter capillary and the converging-diverging flow model with different cross sections are required to describe the flow characteristics of the polymer solution in porous media more accurately. When the polymer solution flows through the port, we have the elongational flow and the polymer molecules undergo elongational deformation elastically. By using the mechanical energy balance principle and the minimum energy principle, a mathematical model of non-Newtonian fluid inlet flow was established by Binding. On the basis of the Binding theory, with the application of the theory of viscoelastic fluid flow in the circular capillary and the contraction - expansion tube, the relations between the viscoelastic fluid flow rate and the pressure drop are obtained.

Key words: Binding theory, viscoelastic fluid, abrupt contraction capillary, permeability rule

Introduction

To accurately describe the constitutive equations of the viscoelastic fluid is the key in this study. In particular, the indoor measurement for the characterization of the elongational viscosity is quite difficult[1,2]. That is why Liang Jizhao applied correction factors as done by Bagley to eliminate the elongational viscosity in his study[3]. Wever et al.[4] presented experimental results of shearing and stretching flows and the first normal-stress difference of the PAM in detail, together with the seepage rule of the polymer solution in four ideal porous media.

When the polymer solution flows in porous media, there exist both the elongational flow and the shear flow during the oil displacement. The additional dissipation due to the elongational deformation is often ignored in the studies of the fluid flow in porous media. For the laminar flow of the Newtonian fluid, this result is supported by experiment. But for the polymer solution the situation is more complex, the elongational pressure drop of the fluid flow through porous media can not to be ignored.

In the capillary of a fixed diameter, the polymer solution is only affected by the shear force, and its rheological property is pseudoplastic. Therefore the variable diameter capillary and the converging-expansion flow model of different cross section shapes are used to describe the flow characteristics of the polymer solution in porous media more accurately. When flowing through these flow channels, the polymer

* Project supported by the National Natural Science Foundation of China (Grant No. 51574085), the Natural Science Founaation of Heilongjiang Province (Grant No. F2015020) and the Science and Technology Research Project of Department of Education of Heilongjiang Province (Grant No. 12521059).
Biography: Er-long Yang (1976-), Male, Ph. D., Professor
Corresponding author: Mei Wang,
E-mail: wmmay@sina.com

solution generates an elongational flow, and the polymer molecules have elongational deformations elastically[5-8]. Under the condition that the elongational viscosity is assumed as a constant, Sienz et al.[9] gave an analytical solution of the pressure drop in the contracting channel by using the cylindrical coordinates and the infinitesimal force balance method[10].

With the principles of the mechanical energy balance and the minimum energy, Binding established a mathematical model of the non-Newtonian fluid inlet flow convergence[11]. Although the original intention of the method is to analyze the extension viscosity of the eupolymer, this method can be used to calculate the relationship between the pressure drop and the flow rate in variable diameter capillaries, under the condition that the elongational and rheological properties of the polymer solution are known and the flow in porous media meets the assumption of macroscopic continuum mechanics.

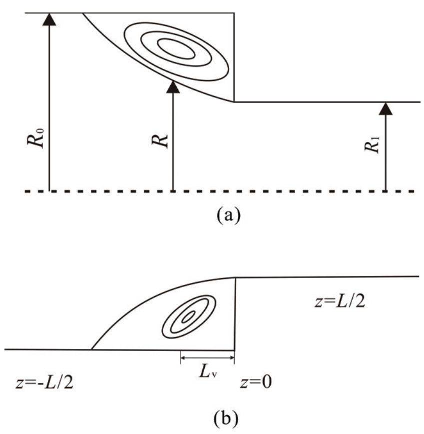

Fig.1 Schematic diagram of abrupt contraction capillary

1. Work of binding

1.1 *Model building*

As shown in Fig.1, the abrupt contraction capillary is a kind of representative porosity models. The capillary length is L, the entrance radius is R_0, the exit radius is R_1, and the ratio of the pore throats is $\beta = R_0 / R_1$. When a non-Newtonian fluid flows from a large cross section channel into a small cross section channel, because of the abrupt contraction of the channel's cross section, the fluid flow line is not parallel and forms a cone boundary. The fluid's natural convergence half angle between the tangent to the boundary line and the center line of the passage is not all equal to the entrance half angle of the passage. When the current angle is less than the latter, before the entrance of the flow channel area, a so-called "circulation area" will be formed, in which the fluid will be in a vortex motion[12]. The circulation of the fluid movement is prone to induce unstable flow phenomenon, and lead to additional energy consumption, a serious concern.

1.2 *Method development*

Assume that the boundary flow line of the convergent flow in the entry of the capillary tube shown in Fig.1 is a curved cone, and is in an axial symmetry. The busbar of the cone can be represented as: $R = f(z)$. When the cylindrical coordinates are used, the velocity field can be expressed as:

$$v_z = \frac{3n+1}{\mathrm{n}+1}\frac{Q}{\pi R^2}\left[1-\left(\frac{r}{R}\right)^{1+1/n}\right] \tag{1}$$

$$v_r = \frac{3n+1}{n+1}\frac{Qr}{\pi R^3}\left[1-\left(\frac{r}{R}\right)^{1+1/n}\right]\frac{\mathrm{d}R}{\mathrm{d}z} \tag{2}$$

The strain rate components are:

$$D_{rr} = \frac{3n+1}{n+1}\frac{Q}{\pi R^3}\left[1-\left(\frac{2n+1}{n}\right)\left(\frac{r}{R}\right)^{1+1/n}\right]\frac{\mathrm{d}R}{\mathrm{d}z} \tag{3}$$

$$D_{\theta\theta} = \frac{3n+1}{n+1}\frac{Q}{\pi R^3}\left[1-\left(\frac{r}{R}\right)^{1+1/n}\right]\frac{\mathrm{d}R}{\mathrm{d}z} \tag{4}$$

$$D_{zz} = \frac{3n+1}{n+1}\frac{Q}{\pi R^3}\left[\frac{3n+1}{n}\left(\frac{r}{R}\right)^{1+1/n}-2\right]\frac{\mathrm{d}R}{\mathrm{d}z} \tag{5}$$

$$D_{rz} = -\frac{3n+1}{n}\frac{Q}{\pi R^3}\left(\frac{r}{R}\right)^{1/n} \tag{6}$$

In Eq.(6), $(\mathrm{d}R/\mathrm{d}z)^2$ and $\mathrm{d}^2R/\mathrm{d}^2z$ are ignored, and the following conditions are satisfied: when $Z=0$, $R=R_1$, if the shear viscosity and the elongational viscosity of the polymer solution obey the power law relationship, according to the principle of the minimum energy, the boundary flow line can be decribed as

$$\left(-\frac{\mathrm{d}R}{\mathrm{d}z}\right)^{t+1} = \frac{K(n+1)^{t+1}}{lt(3n+1)n^n I_{nt}}\left[\frac{(3n+1)Q}{\pi R^3}\right]^{n-t}$$

With the introduction of the Binding's approximate method, the length of the spiral L_v is given as follows

$$\frac{L_v}{2R_1} = \frac{(1+t)}{2\beta(n+1)(2t-3n+1)}\left[\frac{lt(3n+1)n^t I_{nt}}{K}\right]^{1/(1+t)}.$$