Available online at www.sciencedirect.com
ScienceDirect
Journal of Hydrodynamics
2017,29(2):243-250
DOI: 10.1016/S1001-6058(16)60734-5
www.sciencedirect.com/
science/journal/l0016058

A scheme for improving computational efficiency of quasi-two-dimensional model*

Tae Uk Jang[1,2], Yue-bin Wu (伍悦滨)[2,3], Ying Xu (徐莹)[4], Qiang Sun (孙强)[2]
1. *Department of Mechanics, Kim Il Sung University, Pyong Yang, D. P. R. Korea*
2. *School of Municipal and Environmental Engineering, Harbin Institute of Technology, Harbin 150090, China, E-mail: jtu_rns @163.com*
3. *State Key Laboratory of Urban Water Resource and Environment, Harbin Institute of Technology, Harbin 150090, China*
4. *School of Energy and Architecture, Harbin University of Commerce, Harbin 150028, China*

(Received January 7, 2015, Revised November 2, 2015)

Abstract: The quasi-2D model, taking into account the axial velocity profile in the cross section and neglecting the convective term in the 2-D equation, can more accurately simulate the water hammer than the 1-D model using the cross-sectional mean velocity. However, as compared with the 1-D model, the quasi-2D model bears a higher computational burden. In order to improve the computational efficiency, the 1-D method is proposed to be used to solve directly the pressure head and the discharge in the quasi-2D model in this paper, based on the fact that the pressure head obtained as the solution of the two-dimensional characteristic equation is identical to that solved by the 1-D characteristic equations. The proposed scheme solves directly the 1-D characteristic equations for the pressure head and the discharge using the MOC and solves the 2-D characteristic equation for the axial velocities in order to calculate the wall shear stress. If the radial velocity is needed, it can be evaluated easily by an explicit equation derived from the explicit 2-D characteristic equation. In the numerical test, the accuracy and the efficiency of the proposed scheme are compared with two existing quasi-two-dimensional models using the MOC. It is shown that the proposed scheme has the same accuracy as the two quasi-2D models, but requires less computational time. Therefore, it is efficient to use the proposed scheme to simulate the 2-D water hammer flows.

Key words: Water hammer, method of characteristics, numerical scheme, pipe, quasi-2D model

Introduction

The water hammer is a widespread phenomenon in water supply pipeline systems, which often poses a threat to the safety of the pipeline systems, so it is important to simulate the water hammer at the design stage as well as during the operation of the water supply networks[1-3]. Due to easy programming and high computational efficiency, the 1-D model is widely used for the analysis of water hammer problems[4]. However, the 1-D model underestimates the frictional resistances by using a steady or quasi-steady friction term[5-7]. In fact, the velocity gradients at the wall of the pipe are greater in the unsteady flows and thus, the wall shear stresses are larger than the corresponding values in the steady flows[8]. In order to simulate the water hammer flows accurately, a 2-D model or a quasi-2D model should be used. The quasi-2D model[9,10], based on the assumption that the flow is axially symmetric and the convective terms are negligible, associates the 1-D pressure distribution with the 2-D velocity distribution. Because the velocity profiles are taken account of in the cross section, the quasi-2D model simulates the water hammer more accurately than the 1-D model. However, it bears a higher com-

* Project supported by the National Natural Science Fund in China (Grant No. 51208160), the Foundation for Distinguished Young Talents in Higher Education of Heilongjiang Province (Grant No. UNPYSCT-2015072) and the Harbin Science and Technology Project.
Biography: Tae Uk Jang (1962-), Male, Ph. D., Associate Professor
Corresponding author: Yue-bin Wu, E-mail: ybwu@hit.edu.cn

putational burden. Therefore, it is necessary to improve the computational efficiency of the quasi-2D model.

Several numerical schemes were applied to quasi-2D models for analysis of water hammer problems. Vardy and Hwang[11] solved the hyperbolic part of 2-D governing equations by the method of characteristics (MOC) and the parabolic part by finite difference (FD). The model of Vardy and Hwang is known to be accurate and stable[12,13], however, it involves the inversion of a large matrix. Silva-Araya and Chaudhry[14] proposed a scheme to solve the governing equations in the 1-D framework by MOC and the 2-D momentum equation by FD. Pezzinga[5,15] used the explicit FD scheme to solve the 1-D continuity equation and the implicit FD scheme to solve the 2-D momentum equation. The model is efficient due to the decoupling between the 1-D continuity and 2-D momentum equations, through ignoring the radial velocity in the calculation process and evaluating the discharge by numerical integration. Zhao and Ghidaoui[16] proposed an efficient quasi-2D model to improve the numerical efficiency in the model of Vardy and Hwang, which requires the calculation of two smaller tri-diagonal matrices. The model of Zhao and Ghidaoui, with the consideration of the effect of the radial velocity component, is a stable implicit scheme. Wahba[17] proposed a scheme to solve the 1-D continuity equation and the 2-D momentum equation by FD for analysis of water hammer flows in the low Reynolds number range. Korbar et al.[18,19] proposed an efficient scheme to improve the numerical efficiency of the Zhao and Ghidaoui model. In this model, the axial velocity was evaluated using the 2-D characteristic equation and the pressure head was calculated using the 1-D characteristic equation.

This paper proposes to solve directly the pressure head and the discharge in the 1-D form and the wall shear stress in the 2-D form by MOC, based on the fact that the pressure head obtained by the 2-D characteristic equation is identical to that by the 1-D characteristic equations. The accuracy and the efficiency of the proposed scheme are shown by comparing with the two quasi-2D models (i.e., the model of Zhao and Ghidaoui and the model of Korbar et al.) in a numerical test.

1. Governing equations and numerical scheme

1.1 *Governing equations of quasi-2D model*

Assuming that the flow is axially symmetric and the convective terms are negligible, the 2-D governing equations for water hammer flows in a pipe are expressed as follows[16]:

$$\frac{g}{a^2}\frac{\partial H}{\partial t}+\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial (rv)}{\partial r}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+g\frac{\partial H}{\partial x}-\frac{1}{\rho r}\frac{\partial (r\tau)}{\partial r}=0 \tag{2}$$

where x is the axial distance along the pipe, r is the radial distance from the pipe axis, t is the time, ρ is the fluid density, H is the pressure head, u and v are the axial and radial velocities, respectively, a is the wave speed, g is the gravitational acceleration, and τ is the total shear stress. The 1-D governing equations are derived by integrating Eq.(1) and Eq.(2) over the pipe section, yielding

$$\frac{gA}{a^2}\frac{\partial H}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{3}$$

$$\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{\pi D\tau_w}{\rho}=0 \tag{4}$$

with

$$Q=\int_0^R 2\pi ru\,\mathrm{d}r\,,\quad \tau_w=-\left(\rho\nu_T\frac{\mathrm{d}u}{\mathrm{d}r}\right)_{r=R}$$

where Q is the discharge, A is the sectional area of the pipe, R and D are the radius and the diameter of the pipe, respectively, ν_T is the total viscosity, τ_w is the wall shear stress. The governing equations are numerically solved by the MOC and hence, the characteristic forms of the 2-D and 1-D governing equations are given as follows:

$$\frac{\mathrm{d}H}{\mathrm{d}t}\pm\frac{a}{g}\frac{\mathrm{d}u}{\mathrm{d}t}+\frac{a^2}{g}\frac{1}{r}\frac{\partial (rv)}{\partial r}\mp\frac{a}{g}\frac{1}{\rho r}\frac{\partial (r\tau)}{\partial r}=0\quad \text{along}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\pm a \tag{5}$$

$$C_a\frac{\mathrm{d}H}{\mathrm{d}t}\pm\frac{\mathrm{d}Q}{\mathrm{d}t}\pm\frac{\pi D\tau_w}{\rho}=0\quad \text{along}\quad \frac{\mathrm{d}x}{\mathrm{d}t}=\pm a \tag{6}$$

where $C_a=gA/a$.

1.2 *Quasi-2D models*

The computational domain for the discretization of Eq.(5) is shown in Fig.1. The pipe length, L, is divided into N_x reaches with a constant length $\Delta x = L/N_x$ in the axial direction. Each computational point in the axial direction of the pipe is discretized into N_r cylinders with varying thickness in the radial direction. In Fig.1(a), r_j and r_{cj} are the coordinates of the