Ocean Engineering 146 (2017) 257–267

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

# A laboratory experiment on the surface and internal waves in two-layered fluids

Donghyeong Ko[a,b,1], Yeunwoo Cho[a,*]

[a] Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-gu, Daejeon, 34141, Republic of Korea
[b] Hyundai Heavy Industry Co., Ltd., 1000, Bangeojinsunhwan-doro, Ulsan, 682-792, Republic of Korea

ARTICLE INFO

*Keywords:*
Surface waves
Internal waves
Two-layered fluids
Barotropic
Baroclinic

ABSTRACT

A laboratory experiment is performed to investigate the generation of periodic waves with small amplitudes in a system of two fluid layers in the presence of the top free surface. A horizontal cylindrical wavemaker with a circular cross section is oscillated in two immiscible fluids, silicon oil and water, and the motions of the free surface and the interface between the two fluids are monitored using laser images, and these results are further confirmed with wave probe measurements. There are two different wave modes depending on the relative phase between the motions of the free surface and the interface; barotropic if they are in phase and baroclinic if they are 180° out of phase. To determine the mode of dominance, the position of the wavemaker (at the free surface, at the interface, and below the interface) and the oscillation frequency are varied and the observed wave profiles are decomposed into barotropic and baroclinic waves. In the high frequency range, irrespective of the wavemaker position, it is observed that barotropic waves are dominant, whereas in the low frequency range, baroclinic waves become more important as the submergence depth of the wavemaker increases.

## 1. Introduction

A steep density gradient is often generated in the oceans due to the abrupt variations of temperature and/or salinity and, then, the density-stratified sea water is often modeled as a system of two layers with different densities. Theoretically, this simplified two-layered system admits two-dimensional plane waves both on the top free surface and the interface between the two fluids, and these waves are referred to as surface and internal waves, respectively. Furthermore, the waves are called barotropic or baroclinic if they are in phase or 180° out of phase with each other, respectively. Recently, due to their highly nonlinear nature, particular attention has been paid to the propagation of baroclinic solitary waves, which have been observed frequently in many field observations (Sandstrom and Elliott, 1984; Apel et al., 1985; Liu et al., 1998). Their generation processes have been also studied extensively for different generation mechanisms such as the interaction of tidal currents and bottom topography (or, equivalently, translating bottom topographic forcing) and surface wind forcing (Goryachkin et al., 1992; Holloway et al., 1997; Simmons et al., 2004; Garrett and Kunze, 2007; Farmer, 1978; Rubenstein, 1994; Stevens et al., 1996). Previous studies have focused on baroclinic waves at the interface, and it is quite common to assume that the top surface is rigid, instead of free. This is valid only when the density variations are small, where the Boussinesq approximation is applicable. In laboratory experiments, the assumption of small density variations is often violated (Koop and Butler, 1981; Michallet and Barthelemy, 1998) and the presence of the top free surface is no longer negligible even for baroclinic waves and the motion of the top free surface has to be included for the complete description of excited waves. Nevertheless, only a limited number of studies that describe waves in a two-layer system with a free surface, in particular, under the action of external forcing, have been reported. Yeung and Nguyen studied steady three-dimensional waves in a two-layer system for a steadily translating source, and they described wave patterns depending on the Froude number and the density difference (Yeung and Nguyen, 1999). The diffraction problem of incident waves scattered by a fixed horizontal cylinder was studied by Linton and Mclver (1995) and Cadby and Linton (2000) for two and three-dimensional waves, respectively. On the other hand, for the radiation problem of forced waves by an oscillating object, Ten & Kashiwagi performed a boundary element method based numerical study of a floating box-shaped body oscillating in two-layer fluids

* Corresponding author.
*E-mail address:* ywoocho@kaist.ac.kr (Y. Cho).
[1] This research was performed while the first author studied at the division of Ocean Systems Engineering (now, merged into the department of Mechanical Engineering), Korea Advanced Institute of Science and Technology.

https://doi.org/10.1016/j.oceaneng.2017.09.060
Received 14 February 2017; Received in revised form 5 August 2017; Accepted 25 September 2017

and validated their numerical results for hydrodynamic coefficients with experimental measurements (Ten and Kashiwagi, 2004). In more detail, they studied on the generation and propagation of linear surface- and internal gravity waves for two different configurations of a vertically oscillating box-shaped wavemaker; in one case, the wavemaker is positioned on the surface of the upper fluid and, in the other case, the wavemkaer is immersed both in the upper and lower fluids. Their experimental settings (upper to lower fluid height ratio, density ratio, and viscosity ratio) are different from ours. Most of other existing experimental works are on the generation and propagation of nonlinear surface- and internal gravity solitary waves by some wave-making mechanisms; vertically oscillating box-shaped wavemaker at the interface between two fluids (Walker, 1973), dam-breaking type wave making (Michallet and Barthelemy, 1998; Kodaira et al., 2016), horizontally translating ship-shaped model moving on the surface of the upper fluid (Mercier et al., 2011). Also, there exist a few experimental works on the generation and propagation of linear surface- and internal gravity–capillary waves in two-layered fluids (Issenmann et al., 2016; Falcon, 2010). In these works, a horizontally oscillating blade-type wavemaker is used, which is immersed both in the upper and lower fluids. In summary, among the aforementioned previous studies, except the work of Ten and Kashiwagi (2004), few experimental studies have been carried out on the generation and subsequent propagation of periodic waves in two-layered fluids. This is the subject of the present paper. In particular, we monitor the amplitude variations of barotropic and baroclinic waves depending on the frequency and the position of a vertically oscillating horizontal cylindrical wavemaker. After introducing linear water wave theory in two-layer fluids, the experimental set-up and the measurement technique are described. Together with an analysis of the decomposition of observed wave profiles into barotropic and baroclinic modes, experimental results are discussed and future work is also suggested.

## 2. Linear theory

Fig. 1 shows the schematic illustration of surface and internal waves in two-layer fluids with present experimental setting. Here, we consider only gravity waves and neglect the effect of surface tension. The reference coordinate is located at the interface between two still fluids, and the $x$-axis is the direction of the wave propagation and the $z$-axis is the vertical direction in which the gravitational force acts in the negative $z$ direction. The densities of the upper and lower fluids are $\rho_1$ and $\rho_2$, respectively, and the depths of the upper and lower fluids are $h_1$ and $h_2$, respectively. Assuming the two fluids are inviscid and incompressible and the flow is irrotational, the upper-layer velocity potential $\phi_1$ and the lower-layer velocity potential $\phi_2$ satisfying the Laplace equations can be written in complex forms, with the implication of taking the real part, as

$$\phi_1 = \{A \cosh k(z-h_1) + B \sinh k(z-h_1)\}e^{i(kx-\omega t)} \tag{1}$$

$$\phi_2 = \{C \cosh k(z+h_2) + D \sinh k(z+h_2)\}e^{i(kx-\omega t)} \tag{2}$$

Here, $k$ is the wavenumber, $\omega$ is the angular frequency, $t$ is time, and $A, B, C, D$ are unknown coefficients. The displacements of the free surface and the interface are given, respectively, by

$$\zeta = \zeta_0 e^{i(kx-\omega t)} \tag{3}$$

$$\eta = \eta_0 e^{i(kx-\omega t)} \tag{4}$$

where $\zeta_0$ and $\eta_0$ are the amplitudes of surface and internal waves, respectively. These velocity potentials and displacements satisfy the following six linearized boundary conditions. At the top free surface, the kinematic and dynamic boundary conditions are given by

$$\frac{\partial \zeta}{\partial t} = \frac{\partial \phi_1}{\partial z}, \quad \zeta = -\frac{1}{g}\frac{\partial \phi_1}{\partial t} \quad \text{at } z = h_1 \tag{5}$$

At the interface, the two kinematic and one dynamic boundary conditions are given by

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi_1}{\partial z}, \quad \frac{\partial \eta}{\partial t} = \frac{\partial \phi_2}{\partial z}, \quad \rho_1\left(g\eta + \frac{\partial \phi_1}{\partial t}\right) = \rho_2\left(g\eta + \frac{\partial \phi_2}{\partial t}\right) \quad \text{at } z = 0 \tag{6}$$

At the bottom, the boundary condition is given by

$$\frac{\partial \phi_2}{\partial z} = 0 \quad \text{at } z = -h_2 \tag{7}$$

By substituting Eqs. (1)–(4) into Eqs. (5)–(7), one can find that

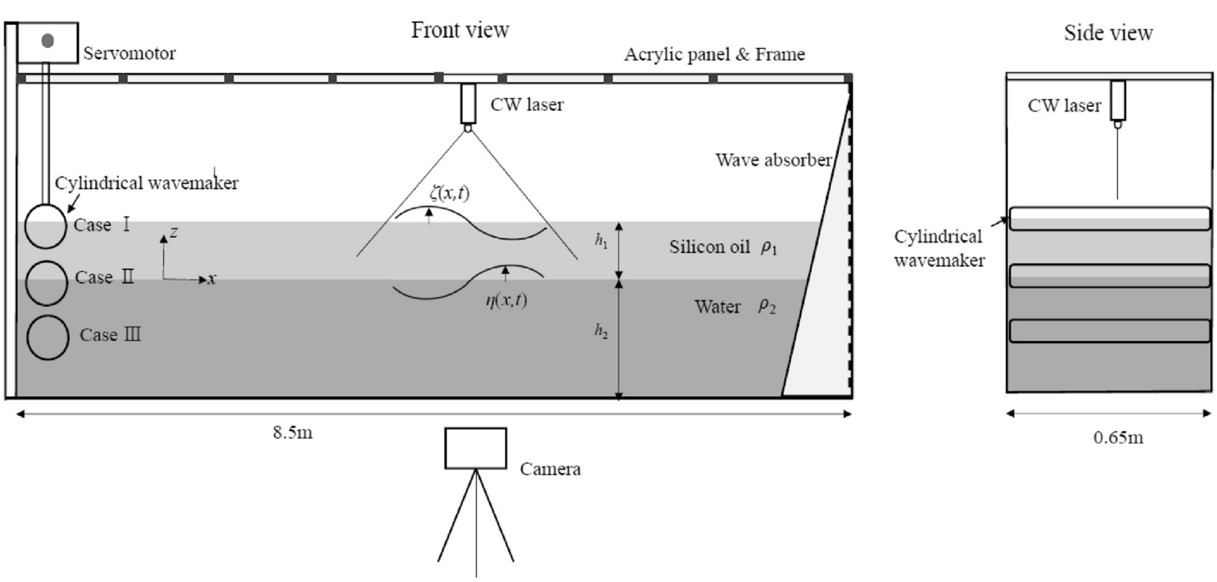


**Fig. 1.** A schematic of the problem: The surface and internal waves are denoted by $\zeta$ and $\eta$, respectively, and both consist of barotropic and baroclinic wave components.