Ocean Engineering 145 (2017) 34–43

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

A combined boundary-finite element procedure for dynamic analysis of plates with fluid and foundation interaction considering free surface effect

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ARTICLE INFO

Keywords:
Fluid-plate-foundation interaction
Hydroelasticity
Fluid storage tanks
Pasternak foundation
Added mass
Free surface
Boundary elements
Mixed finite elements
Elastic bottom
Mindlin plate
Circular/elliptical plate
Free vibration

ABSTRACT

This study combines a mixed finite element formulation and a boundary element procedure for the free vibration analysis of Mindlin plates resting on Pasternak foundation and interacting with a quiescent fluid with a free surface on the other side. The plate-foundation system is represented by a two field mixed finite element formulation, based on the Hellinger-Reissner variational principle, and the fluid-structure interaction is incorporated into the analysis through a boundary element solution. The proposed formulation provides the added mass matrix in terms of the plate deflection, which is appended into the plate equation of motion. The method is applied to the free vibration problem of circular and elliptical bottom plates of rigid fluid storage tanks supported by elastic foundation. Effects of system parameters, such as thickness to width and fluid depth ratios, foundation parameters and plate ellipticity, are extensively studied.

1. Introduction

As a fundamental structural element, plates are extensively used in many engineering disciplines, e.g. as floor and foundation slabs in civil engineering, as bulkhead of a ship in marine engineering (Szilard, 2004), as upper and lower skins of wing in aerospace engineering, etc. In many situations, plates are in contact with external continuums, such as elastic foundation (Kutlu and Omurtag, 2012), fluid medium (Ergin and Uğurlu, 2003), or both (Kutlu et al., 2012), where the mechanical behavior of the plate is considerably altered. Effect of such interactions on plate dynamic performance has been studied by many researchers. Some pioneering attempts regarding the finite element analysis for the free vibration problem of Kirchhoff plate-Pasternak foundation interaction is due to Omurtag et al. (1997) and Omurtag and Kadıoğlu (1998). A comprehensive review of the early literature was reported by Wang et al. (2005), considering the studies on the beam-foundation and plate-foundation interactions. Zhou et al. (2006) employed the three dimensional elasticity equations with small deformation assumption to investigate the free vibration of thick circular plates resting on Pasternak foundation. Akhavan et al. (2009) derived exact-closed form solutions for the free vibration problem of rectangular Mindlin plates resting on Winkler/Pasternak type elastic foundation under the action of in-plane forces. A collocation method incorporated with radial basis functions is proposed by Ferreira et al. (2010) to obtain vibration characteristics of shear deformable plates lying on Pasternak foundation. Finite element and differential quadrature methods are combined by Dehghan and Baradaran (2011), in order to elaborate buckling and free vibration behavior of rectangular thick plates lying on Pasternak foundation, according to the 3-D elasticity theory. Focusing on the studies about plate structures interacting with fluid over the last decade, Ergin and Uğurlu (2003) developed a boundary element solution procedure to estimate the free vibration responses of clamped rectangular plates partially submerged in fluid domain. In a following work, a boundary integral equation solution was carried out by Ergin and Uğurlu (2004) for the free vibration analysis of partially filled fluid storage tanks. A combined Fourier-Bessel series expansion and Rayleigh-Ritz method was suggested by Jeong (2003) to obtain the wet frequency parameters of two identical circular plates confining a fluid domain along with the lateral rigid walls. A similar study considering rectangular plates is performed by Jeong and Kim (2009) by employing the Rayleigh-Ritz method. Askari et al. (2013) examined the free vibration characteristics of circular plates immersed in fluids both analytically and experimentally. Askari and Daneshmand

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http://dx.doi.org/10.1016/j.oceaneng.2017.08.052
Received 10 August 2016; Received in revised form 21 August 2017; Accepted 24 August 2017
Available online 9 September 2017

(2010) analytically investigated the dynamic characteristics of the bottom plate of a fluid filled rigid cylindrical tank, where a rigid cylindrical body is immersed concentrically and partially into the fluid. Kwak and Yang (2013) performed the free vibration analysis of a thin rectangular clamped plate partially submerged in fluid by adopting Mathieu functions to obtain the virtual added mass matrix. Hasheminejad and Tafani (2014) obtained free vibration characteristic of elliptical bottom plates of rigid cylindrical tanks partially filled with inviscid and incompressible fluid. Recently, Ugurlu (2015) conducted a dual reciprocity boundary element solution for the free vibration problem of Kirchhoff plates interacting with fluid.

In spite of the vast literature on the plate-fluid and plate-foundation coupling problems, limited studies have been reported on the mechanical behavior of the plate-fluid-foundation interaction. In a relatively early study, Chiba (1994) solved the axisymmetric vibration problem of a thin elastic bottom plate of a cylindrical liquid container placed on Winkler foundation, where free surface effect was taken into account. Ugurlu et al. (2008) proposed a mixed finite element-boundary element solution procedure to compute the free vibration characteristics of rectangular thin plates resting on Pasternak foundation and completely or partially interacting with an unconfined fluid domain on the other side. Hosseini Hashemi et al. (2010a) studied the same problem by adopting the Mindlin plate and applying the Rayleigh-Ritz method through Timoshenko beam functions; the fluid domain here is considered as limited along depth and width, but infinite in the length direction. Later on, Hosseini Hashemi et al. (2010b) calculated the buckling loads and natural frequencies of the elastic bottom of a rigid rectangular fluid storage tank by also considering the linearly varying in-plane loads. Kutlu et al. (2012) enhanced their physical model in Ugurlu et al. (2008) by substituting thin plate model with a shear deformable plate and replacing the isotropic foundation with a newly proposed orthotropic three parameter foundation. Shahbaztabar and Ranji (2016) employed Rayleigh-Ritz method through Chebyshev polynomials to examine the influence of uniform in-plane loads on dynamic response of symmetrically cross-ply laminated composite plates resting on Pasternak foundation and vertically in contact with fluid based on the Mindlin plate theory. Ugurlu (2016) adopted a higher order boundary element solution procedure to predict the free vibration characteristics of elastic bottom plates of fluid storage tanks resting on Pasternak foundation. Very recently Hasheminejad and Mohammadi (2017) made an attempt to extend the study of Ugurlu (2016) one step further by introducing active control applications by modeling the system through a semi-analytical description of 3D dynamic response characteristics.

The current study adopts a combined numerical analysis approach for the dynamic analysis of plate structures interacting concurrently with elastic foundation and quiescent fluid and attempts to present some new solutions and findings to the literature. It is based on the corporation of a mixed finite element formulation (for the structural problem) and a boundary element scheme (for the fluid problem). The main difference of the proposed method with respect to the previous studies by the authors (Kutlu et al., 2012; Ugurlu et al., 2008) is the inclusion of the free surface effect in the fluid formulation. Mixed formulation of plate-foundation system which was depending on Gâteaux differential is replaced by Hellinger-Reissner principle; so the first variation of the functional is obtained immediately. Additionally, the study avoids the expansion of the fluid velocity potential in terms of modal coordinates, which imposes a weak coupling between the plate and fluid; it offers instead, to define the fluid loading in terms of plate deflection and place it as an added mass directly into the plate equation of motion. According to this approach, once the added mass coefficients are computed for a particular fluid domain, then parametric analysis can be performed for different structural configurations without solving the fluid problem over again. A thorough numerical investigation is performed to expose the features of the coupled solution method and provide an understanding of the dynamic characteristics of plate-foundation system interacting with fluid. Regarding numerical applications; circular and elliptical elastic bottom plates of rigid fluid storage tanks supported by elastic foundation are studied. Unlike Chiba (1994), shear effects in both plate structure and elastic foundation are taken into account by employing the Mindlin plate and Pasternak foundation, respectively. Additionally, vibration characteristics other than axisymmetric modes of the plates are also presented, which were not addressed by Chiba (1994). Since lowest modes are more influential to describe the response of a structure under dynamic loading, first modes other than axisymmetric modes are also important in the modal analysis, as in the case of earthquake excitation. Discovering the lower mode shapes and frequencies shows how the structure inclined to amplify the effect of a load. Choosing an appropriate position to place a machine resting on the structure while reducing the vibrational excitation due to the machine depends on the analysis of the first few influential modes of the structure. They are also used in some other modal analysis based studies such as structural health monitoring and control of structures. Moreover, fatigue analyses of plate type support structures that experience cycling forces of varying frequency, amplitude, and direction, as used in the offshore engineering, involves mode shapes within wider spectrums. In an attempt to examine the influence of key controlling parameters on the system response, a sensitivity study is conducted under different structural, foundation and fluid conditions, i.e., foundation parameters, tank filling ratio, plate thickness and plate ellipticity.

2. Mathematical formulation

2.1. Mixed finite element formulation of the plate-foundation interaction

2.1.1. Field equations of the plate-foundation interaction

Mindlin plate theory describes the displacement field of a moderately thick plate with one transverse translation $\mathbf{U}(w)$ and two independent cross sectional rotations $\mathbf{R}(\varphi_x, \varphi_y)$ (Reddy, 2006). According to the first order shear deformation theory, the deformation field consists of curvatures and constant transverse shear strains, as $\boldsymbol{\varepsilon}(\kappa_x, \kappa_y, \kappa_{xy}, \gamma_{xz}, \gamma_{yz})$ in Cartesian coordinate system (x, y, z) shown in Fig. 1a. In order to construct the Hellinger-Reissner variational formulation, deformation field of the Mindlin plate is expressed in terms of kinematic $(\boldsymbol{\varepsilon}^{\mathbf{u}})$ and constitutive $(\boldsymbol{\varepsilon}^{\boldsymbol{\sigma}})$ relations. Then, the relation $\boldsymbol{\varepsilon}^{\mathbf{u}} = \boldsymbol{\varepsilon}^{\boldsymbol{\sigma}}$ yields,

$$\left.\begin{array}{lll} \kappa_x = \varphi_{x,x} = \dfrac{12}{Eh^3}\left(M_{xx} - M_{yy}\upsilon\right) & , & \kappa_y = \varphi_{y,y} = \dfrac{12}{Eh^3}\left(M_{yy} - M_{xx}\upsilon\right) \\ \kappa_{xy} = \varphi_{x,y} + \varphi_{y,x} = \dfrac{12M_{xy}}{Gh^3} & , & \gamma_{xz} = w_{,x} + \varphi_x = \dfrac{6S_{xz}}{5Gh} \\ \gamma_{yz} = w_{,y} + \varphi_y = \dfrac{6S_{yz}}{5Gh} & , & \end{array}\right\} \tag{1}$$

Here, E, G, υ, and h are the elasticity modulus, shear modulus, Poisson's ratio and plate thickness, respectively. By definition, $\mathbf{M}(M_{xx}, M_{yy}, M_{xy})$ are the moments and $\mathbf{S}(S_{xz}, S_{yz})$ are the transverse shear forces. The stress resultants with their positive directions are displayed in Fig. 1b.

Pasternak mechanical foundation model defines the force intensity between plate and foundation (see Fig. 1b) as $p_f = kw - G_f(w_{,xx} + w_{,yy})$, where k is the Winkler foundation parameter and G_f is the shear foundation parameter. The equilibrium equations of Mindlin plate interacting with Pasternak foundation are given as

$$\left.\begin{array}{c} S_{xz,x} + S_{yz,y} - kw + G_f\left(w_{,xx} + w_{,yy}\right) = 0 \\ M_{xx,x} + M_{xy,y} - S_{xz} = 0 \\ M_{yy,y} + M_{xy,x} - S_{yz} = 0 \end{array}\right\} \tag{2}$$

Here the external loads are avoided.

2.1.2. First variation of the two field functional

Referring to the Hellinger-Reissner variational principle (Hellinger, 1914; Reissner, 1950), the first variation of a two field functional can be