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An efficient 3D non-hydrostatic model for simulating near-shore breaking waves

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ABSTRACT

This paper describes the formulation and verification of a 3D non-hydrostatic free surface water flow model with a Finite Volume Method (FVM) on unstructured grids. The model extends the Non-linear Shallow Water (NSW) equations by utilizing a non-hydrostatic pressure term for describing dispersive water waves. In order to simulate the near-shore wave dynamics, the wave breaking is modeled using a so-called splitting algorithm in the present model. When a wave is ready to break, the non-hydrostatic model is locally switched to the hydrostatic model to suppress the dispersive effects. The breaking wave front is analogy to a moving hydraulic jump, and is treated as a shock with its energy dissipation implicitly evaluated according to the shock-wave theory. The wave breaking is then simulated by solving the conventional NSW with a 2nd order Total Variation Diminishing (TVD) scheme. Extensive case studies are carried out to verify the efficiency and accuracy of the model. The numerical simulations are found to agree well with the experimental measurements.

1. Introduction

Many flows in natural environments are confined between a solid bed beneath and a free surface above the water body. These flows are common in oceanic shelves, estuaries, and rivers, and they are generally referred to as shallow water flows (Borthwick and Barber, 1992; Fringer et al., 2006; Liang et al., 2006; Zhang and Liu, 2009). Such flows are traditionally described by the Non-linear Shallow Water equations (NSW). Researchers have made significant progress to solve this kind of models. The efficient solutions of the NSW enable the modeling of large-scale long wave dynamics, such as tides, storm surges, and tsunamis. Despite the advancements, the difficulties in connection with breaking and dispersive waves in the near-shore region remain unresolved.

To improve the handling of dispersive water waves, the NSW has been developed to account for the non-hydrostatic pressure distribution. Casulli (1999) and Casulli and Zanolli (2002) reported a fully-hydrodynamic pressure treatment that expresses the pressure as a sum of the hydrostatic and non-hydrostatic constituents. Jankowski (1999) presented the details of a predictor-corrector method for the calculation of the pressure field. Such a non-hydrostatic model has been widely implemented in the Cartesian coordinate system (Li and Fleming, 2001; Kocyigit et al., 2002; Chen, 2003) and the unstructured grid system (Fringer et al., 2006). The models of this kind have been used to simulate dispersive waves and flows in the domains with complex geometries (Zhang et al., 2014).

The non-hydrostatic model enables the expansion of the application of the NSW from long waves to relatively short waves, i.e. from non-dispersive waves to dispersive waves. However, few applications to the near-shore region have been carried out by this kind of models because of their inability to handle wave breaking. To model wave breaking, the Navier-Stokes equations with Volume of Fluid (VOF) treatment is able to simulate the complex free surface transformation and breakage, but the high computational costs restrict their practical applications involving natural coasts. Besides of the VOF method for capturing the free surface, the Smoothed Particle Hydrodynamics (SPH) model is a powerful, but computationally-demanding, mesh-free technique for simulating free surface flows, with violent breaking waves (Liang, 2010; Pu et al., 2013; Chen et al., 2015). Compared with the highly accurate numerical models capable of representing the full spectrum of wave dynamics, the Non-linear Shallow Water equations (NSW) and Boussinesq-type (BT) models are still more widely used for simulating the near-shore wave dynamics because of their lower computational costs. However, the NSW cannot represent wave dispersion and the BT models cannot handle wave breaking. The earlier methods of modeling the wave breaking with the BT models are by means of introducing an extra wave energy dissipation term in the governing equations (Zelt, 1991; Karambas and Koutitas, 1992; Schäffer et al., 1993; Kennedy

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et al., 2000; Cienfuegos et al., 2010). Recently, a family of fully-nonlinear BT model were proposed for simulating wave breaking with a splitting algorithm (Bonneton et al., 2011b; Tissier et al., 2012). The idea is to switch the numerical model from the BT equations to the NSW equations when the waves are about to break. In this way, the non-physical dispersive behavior of waves at steep water surface gradients is avoided and the mechanical energy dissipation associated with the wave breaking is modeled by the shock theory. Similar ideas have been adopted in non-hydrostatic models. Ma et al. (2012) described a NHWAVE non-hydrostatic numerical model for simulating the wave dynamics in the surf and swash zones, based on a type of combined FVM and FDM on structured grids. The NHWAVE model is extended to simulate breaking waves using shock-capturing schemes. The similar numerical method was also adopted by Pieter et al. (2013, 2014) in a vertical 2D non-hydrostatic model for investigating coastal processes, and by Wei and Jia (2014) in a non-hydrostatic finite element model for studying coastal waves. Coupled with an efficient numerical technique to handle wave breaking, the non-hydrostatic models are expected to be supervisor to the depth-integrated models in modeling coastal wave dynamics.

In this paper, an in-house code named HydroFlow® was developed, which is a 3D non-hydrostatic FVM model on unstructured grids. Similar to the splitting algorithm used in the BT model, the non-hydrostatic equations are replayed by the hydrostatic equations at the local wave front when wave breaking occurs. Several case studies with available experimental data are used to validate the present model's performance in predicting wave transformation, breaking and run-up on beaches.

2. Numerical model

2.1. Governing equations

The non-hydrostatic pressure distribution is implemented in the model by representing the total pressure as a superposition of the hydrostatic component p_h and the non-hydrostatic p_n component (Casulli, 1999). In order to accurately capture the shapes of the free surface and the uneven bottom boundary, the vertical coordinate z is transformed to the σ coordinate, and the transformed equations can be rewritten as:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_\sigma}{\partial \sigma} = 0 \tag{1}$$

$$\frac{\partial q_x}{\partial t} + \frac{\partial q_x u}{\partial x} + \frac{\partial q_x v}{\partial y} + \frac{\partial q_x \widetilde{\omega}}{\partial \sigma} = -gD\frac{\partial \zeta}{\partial x} - \frac{D}{\rho_0}\frac{\partial p_n}{\partial x} + fq_y + \frac{\partial}{\partial x}\left(\upsilon_t \frac{\partial q_x}{\partial x}\right) + \frac{\partial}{\partial y}\left(\upsilon_t \frac{\partial q_x}{\partial y}\right) + \frac{1}{D}\frac{\partial}{\partial \sigma}\left(\frac{\upsilon_t}{D}\frac{\partial q_x}{\partial \sigma}\right) \tag{2}$$

$$\frac{\partial q_y}{\partial t} + \frac{\partial q_y u}{\partial x} + \frac{\partial q_y v}{\partial y} + \frac{\partial q_y \widetilde{\omega}}{\partial \sigma} = -gD\frac{\partial \zeta}{\partial y} - \frac{D}{\rho_0}\frac{\partial p_n}{\partial y} - fq_x + \frac{\partial}{\partial x}\left(\upsilon_t \frac{\partial q_y}{\partial x}\right) + \frac{\partial}{\partial y}\left(\upsilon_t \frac{\partial q_y}{\partial y}\right) + \frac{1}{D}\frac{\partial}{\partial \sigma}\left(\frac{\upsilon_t}{D}\frac{\partial q_y}{\partial \sigma}\right) \tag{3}$$

$$\frac{\partial q_z}{\partial t} + \frac{\partial q_z u}{\partial x} + \frac{\partial q_z v}{\partial y} + \frac{\partial q_z \widetilde{\omega}}{\partial \sigma} = -\frac{1}{\rho_0}\frac{\partial p_n}{\partial \sigma} + \frac{\partial}{\partial x}\left(\upsilon_t \frac{\partial q_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(\upsilon_t \frac{\partial q_z}{\partial y}\right) + \frac{1}{D}\frac{\partial}{\partial \sigma}\left(\frac{\upsilon_t}{D}\frac{\partial q_z}{\partial \sigma}\right) \tag{4}$$

where $q_x = Du$, $q_y = Dv$, $q_z = Dw$, $q_\sigma = D\widetilde{\omega}$, u, v, w are the velocities in x, y, z directions, respectively, and the velocity along the σ coordinate is calculated as:

$$q_\sigma = \frac{q_z}{D} - \frac{q_x}{D}\left(\sigma\frac{\partial D}{\partial x} + \frac{\partial \zeta}{\partial x}\right) - \frac{q_y}{D}\left(\sigma\frac{\partial D}{\partial y} + \frac{\partial \zeta}{\partial y}\right) - \left(\sigma\frac{\partial D}{\partial t} + \frac{\partial \zeta}{\partial t}\right) \tag{5}$$

In the above equations, g is gravitational acceleration, $f = 2\omega \sin\phi$ is the Coriolis force coefficient, ϕ is the latitude, ω is the Earth's angular speed, ζ is the free surface elevation, h is the still water depth, and $D = h + \zeta$ is the total water depth. When the non-hydrostatic pressure p_n is ignored, then the vertical momentum equation should be also neglected. Then, the system described by Eqs. (1)–(4) degenerates to the conventional shallow water model that consists of only the horizontal momentum equations and the continuity equation.

2.2. Turbulence model

To close the system, the eddy viscosity concept is adopted in the present study and the eddy viscosity coefficient υ_t is determined using the one-equation Spalart-Allmaras (SA) model (Spalart et al., 2000). The transport equation for $\tilde{\upsilon}$ is:

$$\frac{D\tilde{\upsilon}}{Dt} = c_{b1}\tilde{S}\tilde{\upsilon} - c_{w1}f_w\left(\frac{\tilde{\upsilon}}{d}\right)^2 + \frac{1}{\sigma}\{\nabla\cdot[(\upsilon + \tilde{\upsilon})\nabla\tilde{\upsilon}] + c_{b2}(\nabla\tilde{\upsilon})^2\} \tag{6}$$

where

$$\chi \equiv \frac{\tilde{\upsilon}}{\upsilon}, f_w = g_1\left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6}\right]^{1/6}, g_1 = r + c_{w2}(r^6 - r), r \equiv \frac{\tilde{\upsilon}}{\tilde{S}\kappa^2 d^2}, \tilde{S} = |\bar{S}| + \frac{\tilde{\upsilon}}{\kappa^2 d^2}f_{v2}, f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}, f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}}, \bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right),$$

and d is the distance to the nearest solid wall. The model constants are:

$$c_{b1} = 0.1355, \sigma = 2/3, c_{b2} = 0.622, \kappa = 0.41, c_{w1} = c_{b1}/\kappa^2 + (1 + c_{b2})/\sigma, c_{w2} = 0.3, c_{w3} = 2.0, c_{v1} = 7.1$$

The eddy viscosity υ_t is then computed according to:

$$\upsilon_t = \tilde{\upsilon}f_{v1} \tag{7}$$

2.3. Numerical scheme

The numerical model is based on the multi-layered discretization on unstructured grids and the σ-coordinate transformation in the vertical direction, as shown in Fig. 1. On the cell faces, a non-orthogonal horizontal local coordinate system (ξ, η) is used to replace the global Cartesian coordinates (x, y). The derivatives of a function φ on the cell face are carried out in the local coordinates, which are presented as:

$$\phi_x = \frac{1}{J}(\phi_\xi y_\eta - \phi_\eta y_\xi), \quad \phi_y = \frac{1}{J}(\phi_\eta x_\xi - \phi_\xi x_\eta), \quad J = x_\xi y_\eta - x_\eta y_\xi \tag{8}$$

where ξ is directed from a Control Cell (CC) center to a Neighboring Cell (NC) center across the common face, and η is directed along the common boundary and in the anticlockwise direction around the ξ

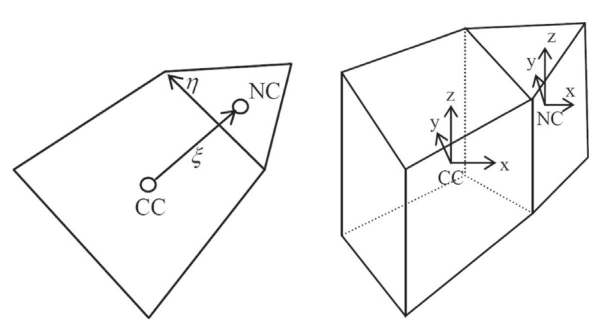

Fig. 1. Local coordinates of the control volume. (a) CC-control volume center (b) NC-neighbor cell center.