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Towards non-intrusive reduced order 3D free surface flow modelling

D. Xiao[a,*,1], F. Fang[a,*], C.C. Pain[a], I.M. Navon[b]

[a] Applied Modelling and Computation Group, Department of Earth Science and Engineering, Imperial College London, Prince Consort Road, London SW7 2BP, UK

[b] Department of Scientific Computing, Florida State University, Tallahassee, FL 32306-4120, USA

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ABSTRACT

In this article, we describe a novel non-intrusive reduction model for three-dimensional (3D) free surface flows. However, in this work we limit the vertical resolution to be a single element. So, although it does resolve some non-hydrostatic effects, it does not examine the application of reduced modelling to full 3D free surface flows, but it is an important step towards 3D modelling. A newly developed non-intrusive reduced order model (NIROM) (Xiao et al., 2015a) has been used in this work. Rather than taking the standard POD approach using the Galerkin projection, a Smolyak sparse grid interpolation method is employed to generate the NIROM. A set of interpolation functions is constructed to calculate the POD coefficients, where the POD coefficients at previous time steps are the inputs of the interpolation function. Therefore, this model is non-intrusive and does not require modifications to the code of the full system and is easy to implement.

By using this new NIROM, we have developed a robust and efficient reduced order model for free surface flows within a 3D unstructured mesh finite element ocean model. What distinguishes the reduced order model developed here from other existing reduced order ocean models is (1) the inclusion of 3D dynamics with a free surface (the 3D computational domain and meshes are changed with the movement of the free surface); (2) the incorporation of wetting-drying; and (3) the first implementation of non-intrusive reduced order method in ocean modelling. Most importantly, the change of the computational domain with the free surface movement is taken into account in reduced order modelling. The accuracy and predictive capability of the new non-intrusive free surface flow ROM have been evaluated in Balzano and Okushiri tsunami test cases. This is the first step towards 3D reduced order modelling in realistic ocean cases. Results obtained show that the accuracy of free surface problems relative to the high fidelity model is maintained in ROM whilst the CPU time is reduced by several orders of magnitude.

1. Introduction

The numerical simulation of ocean modelling is important to a wide range of applications such as atmosphere, sea ice, climate prediction, biospheric management and especially natural disasters (for example, flood and tsunami). The natural disasters often cause big losses and tragic consequences. In order to reduce the losses, a real-time, early-warning and rapid assessment model is required. In comparison to 2D modelling, 3D ocean modelling provides better understanding and much more information about local flow structures, vertical inertia, water level changes, unsteady dynamic loads on structure interacting with fluids, flow structures close to islands and dikes etc. However, the majority of existing 3D ocean models suffer from an intensive computational cost and cannot respond rapidly for tsunami forecasting. In this case, model reduction technology has been presented to mitigate the expensive CPU computational cost since the model reduction technology offers the potential to simulate complex systems with substantially increased computation efficiency.

Among existing model reduction techniques, the proper orthogonal decomposition (POD) method has proven to be an efficient means of deriving the reduced basis functions for high-dimensional nonlinear flow systems. The POD method and its variants have been successfully applied to a number of research fields, for example, signal analysis and pattern recognition (Fukunaga, 1990), statistics (Pearson, 1901), geophysical fluid dynamics and meteorology (Crommelin and Majda, 2004), ocean modelling (Xiao et al., 2013, 2015a, 2015b; Cao et al., 2007), large-scale dynamical systems (Antoulas, 2005), ecosystem modelling (Pelc et al., 2012), data assimilation of wave modelling (Wahle et al., 2015; Altaf et al., 2015), ground-water flow (Vermeulen et al., 2004), air pollution modelling (Fang et al., 2014), shape

* Corresponding author.
E-mail addresses: dh.xiao@imperial.ac.uk (D. Xiao), f.fang@imperial.ac.uk (F. Fang).
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optimisation (Diez et al., 2015), aerospace design (Manzoni et al., 2015; Iuliano and Quagliarella, 2013), lithium-ion batteries convective Boussinesq flows (San and Borggaard, 2015), mesh optimisation model (Fang et al., 2010) and also shallow water equations. This includes the work of Stefanescu and Navon (2013); Stefanescu et al. (2014); Daescu and Navon (2008); Bistrian and Navon (2015); Chen et al., (2011, 2012); Du et al. (2013) as well as Fang et al., (2013, 2009b).

However, the standard reduced order modelling is usually generated through POD and Galerkin projection method, which means it suffers from instability and non-linearity efficiency problems. Various methods for improving numerical instability have been developed such as regularisation method (Jafarpour and Feriedoun, 2012), Xiao et al. (2013); Fang et al. (2013), method of introducing a diffusion term (Bou-Mosleh et al., 2011; Serpas et al., 2011) and Fourier expansion (Willcox and Megretski, 2003). For non-linear efficiency problems, a number of methods have been proposed including empirical interpolation method (EIM) (Barrault et al., 2004) and discrete empirical interpolation method (DEIM) (Chaturantabut and Sorensen, 2010), residual DEIM (RDEIM) (Xiao et al., 2014), Gauss-Newton with approximated tensors (GNAT) method (Carlberg et al., 2013), least squares Petrov-Galerkin projection method (Bou-Mosleh et al., 2011), and quadratic expansion method (J Du et al., 2013; Juan Du et al., 2013; Fang et al., 2009b).

However, those methods are still dependent on the full model source codes. In many contexts, the source codes governed by partial differential equations need to be modified and maintained. Developing and maintaining these modifications are cumbersome (Chen, 2012). To circumvent these shortcomings, non-intrusive approaches have been introduced into ROMs. Chen presented a black-box stencil interpolation non-intrusive method (BSIM) based on machine learning methods (Chen, 2012). D. Wirtz et al. proposed the kernel methods where the learning methods are based on both support vector machines and a vectorial kernel greedy algorithm (Wirtz et al., 2013; Wirtz and Haasdonk, 2012). Audouze et al. proposed a non-intrusive reduced order modelling method for nonlinear parametrized time-dependent PDEs using the radial basis function approach and POD (Audouze et al., 2013, 2009). Klie used a three-layer radial basis function neural network combined with POD/DEIM to predict the production of petroleum reservoirs (Klie et al., 2013). Walton et al. developed a NIROM for unsteady fluid flows using the radial basis function (RBF) interpolation and POD (Walton et al., 2013). Noori et al. (2013) and Noack et al. (2011) applied a neural network to construct the ROM. Xiao et al. presented a non-intrusive reduced order modelling method for Navier-Stokes equations based on POD and the RBF interpolation (Xiao et al., 2015b) and applied it successfully into fluid-structure interaction problems (Xiao et al., 2016, 2017). The CPU computational times are reduced by several orders of magnitude by using this POD-RBF method. Xiao et al. also introduced the Smolyak sparse grid interpolation method into model reduction to construct the NIROM (Xiao et al., 2015a).

POD ROM approaches have been applied to ocean problems (Fang et al., 2009a, 2009b; Ha et al., 2008; Zokagoa and Soulaïmani, 2012). Ha et al. introduced ROM into tsunami forecasting (Ha et al., 2008), and Zokagoa and Soulaïmani (2012) used POD/ROM for Monte-Carlo-type applications. In their work, the POD-based reduced-order models were constructed for the shallow water equations. In shallow water modelling, however there are some errors in results when involving ocean problems like radical topography changes, short waves and local flows around the buildings or mountains. The work of Fang et al. (2009a, 2009b); Du et al. (2013), and Xiao et al. (2013) introduced POD ROM for 2D/3D Navier-Stokes unstructured mesh finite element fluid modelling. However 3D free surface flow examples were not included in their work due to the difficulty in implementation of intrusive POD-ROMs. The implementation difficulty was caused by the change of both the computational domain and 3D unstructured meshes with free surface movement. However, NIROM is capable of handling this issue easily.

This paper, for the first time, constructs a NIROM for free surface flows within the framework of an unstructured mesh finite element ocean model. This is achieved by using the Smolyak sparse grid interpolation method. The Smolyak sparse grid method is a widely used interpolation method and is used to overcome the curse of dimensionality. It was also used for uncertainty quantification for electromagnetic devices (Sumant et al., 2012) where the Smolyak sparse grid was used to calculate statistically varying material and geometric parameters which were the inputs of the ROM. Xiao et al. first used Smolyak sparse grids to construct ROM (Xiao et al., 2015a) and it has been shown to be a promising non-intrusive method for representing complex physical system using a set of hyper-surface interpolating functions. The NIROM can be treated as a black box, which uses a set of hypersurfaces constructed based on the Smolyak sparse grid collocation method to replace the traditional reduced order model. The errors in the NIROMs come from: the POD function truncation error (the ability of the basis functions to represent the solution), the error associated with having just a certain number of solution snapshots (rather than the solution at all time steps) and the error from the calculation of the NIROM solution (for more details, please see Xiao et al., 2017) using, for example, sparse grids or Radial Basis Functions.

In this work, the newly presented NIROM method based on Smolyak sparse grids is applied to complex ocean free surface flows. The capability of newly developed NIROM for 3D free surface flows are numerically tested and illustrated in Balzano and Okushiri tsunami test cases. The main novelty of this work is the inclusion of 3D flow dynamics with a free surface and the wetting-drying front. The solutions from the full fidelity ocean model are recorded as a sequence of snapshots, and from these snapshots appropriate basis functions are generated that optimally represent the flow dynamics. The Smolyak sparse grid interpolation method is then used to form a hyper-surface that represents the ROM. Once the hyper-surface has been constructed, the POD coefficient at current time step can be obtained by providing the POD coefficients at previous time steps to this hyper-surface. Numerical comparisons between the high fidelity model and this NIROM are made to investigate the accuracy of this novel NIROM for free surface flows.

The structure of the paper is as follows. Section 2 presents the governing equations of free surface flows. Section 3 presents the derivation of the POD model reduction and re-formulation of the governing equations using the Smolyak sparse grid method. Section 4 illustrates the methodology derived above via two numerical examples. This is based on two test problems where the Balzano test case and Okushiri tsunami test case are numerically simulated. Finally in Section 5 conclusions are presented and the novelty of the manuscript is fully summarized and illuminated.

2. Three dimensional governing equations for free surface flows

2.1. 3D Navier-Stokes equations

The three dimensional incompressible Navier-Stokes equations with Boussinesq approximation and the conservative equation of mass are used in this work:

$$\nabla\cdot\vec{u} = 0, \tag{1}$$

$$\frac{\partial\vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u} = -\nabla p + \nabla\cdot\tau. \tag{2}$$

where the terms $\vec{u} \equiv (u_x, u_y, u_z)^T$ are the velocity vector, p the perturbation pressure ($p{:=}p/\rho_0$, ρ_0 is the constant reference density).