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Numerical investigation and dynamic behavior of pipes conveying fluid based on isogeometric analysis

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ABSTRACT

Dynamic analysis and design of slender structures subject to different types of applied forces is of considerable practical interest in ocean engineering and aerospace applications. In this paper, the stability of pipes subjected to influences induced by fluid flow is investigated by using IsoGeometric Analysis (IGA). IGA aims to use the exact Computer-Aided Design (CAD) geometry and reduce the geometrical errors introduced by approximation of the physical domain of the problem. It uses B-Splines and Non-Uniform Rational B-Splines (NURBS) as basis functions and provides several advantages, which make it suitable for efficient and accurate simulations. The mathematical formulation of the problem is developed based on the Euler-Bernoulli beam theory (EBT) and the plug-flow model for the fluid forces. Both divergence and flutter instabilities are investigated. The numerical results are compared with those obtained by other available methods and it is shown that the results are in good agreement with lesser number of degrees of freedom, which is a fundamental criterion for the computational cost of an analysis. It is also confirmed that the IGA as described in the present paper, can be used efficiently to predict the possibility of divergence and coupled-mode flutter in dynamic analysis of pipes conveying fluid.

1. Introduction

The stability of slender structures conveying fluids has been of considerable interest for engineers and designers over the past century. Some engineering applications include oil and hydraulic pipelines, heat exchangers, liquid-fuel piping systems, air-conditioning ducts, offshore piping, etc. (Rafiei et al., 2012; Ibrahim, 2010). A particular area of interest that has received extensive attention over the past few decades is the stability of pipes conveying fluid. The dynamic interaction between the fluid and the pipe causes energy to be transferred to the pipe and the pipe becomes unstable after a sufficient amount of energy exchange. The main reasons for special attention to the problem of pipes conveying fluid are their (i) simplicity and (ii) possible use in understanding the theoretical and experimental response of more complicated systems (Paidoussis, 2014).

Over the past sixty years, dynamics of fluid-conveying pipes have been studied by different analytical, numerical and experimental methods for a wide range of boundary conditions and loadings (Ibrahim, 2010; Aldraihem, 2007; Zhang et al., 2016). Notable contributions to the problem have been done by Paidoussis and his coworkers (Modarres-Sadeghi and Païdoussis, 2009). The dynamic analysis is usually conducted using the Euler-Bernoulli beam theory and different solution methods can be used for vibration analysis of the problem (e.g., Finite Element Method (FEM)). Several finite element models for the fluid-structure interaction analysis of liquid-filled pipes have also been developed by many authors (e.g., Hansson and Sandberg, 2001; Stangl et al., 2009 and Zare et al., 2011).

With the progress of advanced computational tools, more efficient numerical methods can be developed for computation of structural vibrations (Kazemzadeh-Parsi and Daneshmand, 2013; Daneshmand and Kazemzadeh-Parsi, 2009; Kazemzadeh-Parsi and Daneshmand, 2012). Development of such computational tools is normally based on Computer-Aided Design (CAD) whereas the resulting geometries are used in both manufacturing processes and numerical calculations. A new development of FEM has recently been proposed and called Isogeometric Analysis (IGA). IGA is a new simulation technology which uses the same class of basis functions for both representing the geometry of the computational domain and approximating the solution of the problem (Bazargan Lari, 2015). In other words, IGA aims to use the exact CAD geometry and reduce the geometrical errors introduced by approximation of the physical domain of the problem. This paper focuses on the application of IGA in the analysis of transverse vibration of pipes conveying fluid. The method, despite its young age, has been significantly matured as a link between the finite element method and

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computer aided design (Wang et al., 2015; Chen et al., 2014). In spite of the extensive achievements in dynamic analysis of fluid-structure systems, the application of IGA in transverse vibration analysis of pipes conveying fluid has not been reported so far. In this paper, a well-known straight fluid-conveying pipe is modeled based on the Euler-Bernoulli beam theory. Moreover, a short introduction on isogeometric approach is provided and the main ideas behind this new method in computational mechanics are explained. It will be shown that using NURBS-based IGA discretization with higher smoothness improves the solution accuracy and can be considered as a promising approach for analysis of pipes conveying fluid.

The paper is outlined as follows: The theoretical background of IGA is presented in Section 2. In Section 3, dynamical equations of in-plane motion for a pipe conveying fluid is given in details. The isogeometric weak formulation of the transverse vibration problem is also derived in order to be used in developing the discretized equations in the matrix form. In Section 4, the results obtained from the present formulation are given and discussed. In order to validate the IGA computer code, free vibration analysis of a rotation free (RF) Euler-Bernoulli beam problem is considered and the results are compared with the FEM solutions. Moreover, dynamic behavior of a clamped-clamped Euler-Bernoulli pipe is considered and natural frequencies of the system are calculated by using the isogeometric analysis developed in this paper. The results of our calculations are also compared with those available in literature. Finally, the characteristic equation of a pipe conveying fluid is obtained from which the corresponding critical velocity of the fluid flow and natural frequencies of the pipe are determined. The concluding remarks are presented in the last section and the main advantages of using the isogeometric analysis in vibration analysis of pipes conveying fluid are summarized. In summary, it is shown that the implementation of IGA and NURBS in dynamic analysis of pipes conveying fluid results in higher convergence rates and highly accurate results with fewer elements.

2. Theoretical background of isogeometric analysis

Development of IGA is fundamentally due to two important drawbacks in using traditional numerical methods: (i) approximation involved in the geometrical representation of the boundaries of the problem domain, and (ii) time consuming communications needed between the discretized geometry and the analysis tool in order to provide the adaptivity and refinement of the solution (Dedè et al., 2012). IGA uses the functions employed in CAD systems and is capable of representing many engineering geometries exactly. It also simplifies mesh refinement process and provides a unique geometric description for all meshes with all orders of approximation. A primary tool in the establishment of IGA is the non-uniform rational B-Splines (NURBS). NURBS are a standard tool for describing and modeling curves and surfaces with several interesting properties and can be used in design and analysis of a wide range of problems in linear and nonlinear solid and structural mechanics. Since B-Splines are the basis for building NURBS, they are investigated first. Unlike the standard finite element analysis in which the shape functions are defined over the elements, the influence domain of B-Splines is described by patches. Patches are subdomains on which the element types and material behavior are assumed to be uniform. Each element in the physical space (Ω^{ks}) is mapped to its corresponding image element in parameter space ($\widehat{\Omega}^{ks}$). Furthermore, an element in parametric domain may be mapped into a parent element ($\overline{\Omega}^{ks}$) in order to unify integration procedure on elements (Fig. 1). It should be noted that mapping itself is global for the whole patch, rather than for the elements themselves.

The NURBS *basis* functions are usually *not* interpolatory. This is in contrast with the fundamental property of the shape functions in FEM. Two types of meshes are defined in NURBS; control meshes and physical meshes. Control meshes are defined by control points (CPs) and do not necessarily conform to the actual geometry. In fact, they control the geometry. Control variables are located at the CPs and can be considered as degrees of freedom (DOF) (Piegl and Tiller, 1997).

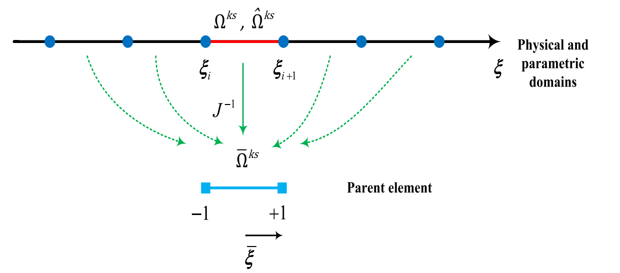

Fig. 1. Geometrical and affine mappings for integration by Gaussian quadrature on NURBS knot spans of pipe.

2.1. Knot vectors

Knot vector is a set of non-decreasing coordinates in parameter space, written as $\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$, where $\xi_i \in \mathbb{R}$ is the i^{th} knot, i is the knot index ($i = 1,2....,n + p + 1$), p is the polynomial order, and n is the number of basis functions utilized to construct a B-Spline curve. Parameter space is divided into elements using knots. Element boundaries in the physical space are simply the images of lines constructed between knots under B-Spline mapping.

Knot vector may be uniform, if the knots are equally spaced in the parameter space, and non-uniform if they are unequally spaced. Knot values may be repeated. Number of the value repetition of a knot is called multiplicity of a knot and plays an important role in implementing basis functions (Cottrell et al., 2009). A knot vector is said to be open if its first and last knot values appear $p + 1$ times. A basis function which is formed from an open knot vector is interpolatory at the ends of the parameter space interval. This is an interesting and distinguishing feature between "knots" in IGA and "nodes" in Finite Element Analysis (FEA) (Juttler et al., 2015).

2.2. Basis functions

B-Spline basis functions are recursively defined by using an available knot vector. The first basis function is written with piecewise constants ($p = 0$) as,

$$N_{i,0}(\xi) = \begin{cases} 1 \text{ if } \xi_i \le \xi < \xi_{i+1} \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

For $p = 1,2,3,...$, the B-Spline basis functions are defined as

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \tag{2}$$

Eq. (2) is referred to as the *Cox–de Boor* recursion formula (Piegl and Tiller, 1997; Hughes et al., 2005).

Among many useful properties of B-Spline functions, the following properties are more interesting:

- Piecewise polynomials of degree p: $N_{i,p}$ $\quad i = 1....,n$.
- Compact support: $\text{supp}(N_{i,p}) = [\xi_i, \xi_{i+p+1})$, $i = 1,...,n$.
- Non-negativity: $N_{i,p}(\xi) \ge 0$, $\forall \xi \in [\xi_1, \xi_r]$, $i = 1,...,n$.
- Partition of unity: For an open knot vector, $\sum_{i=1}^{n} N_{i,p}(\xi) \equiv 1$
- Continuity: The basis functions $N_{i,p}$ are p times continuously differentiable (C^p-continuous) inside a knot span, and at inner knots of multiplicity k ($k \le p$), they are only C^{p-k}.

Some examples of B-Spline functions are shown in Figs. 2 and 3.

Now let $N_{i,p}^{(k)}$ denote the kth derivative of $N_{i,p}(\xi)$. Repeated differentiation of Eq. (2) produces the general formulation as follows (Piegl and Tiller, 1997; Hughes et al., 2005):