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Water-wave diffraction and radiation by multiple three-dimensional bodies over a mild-slope bottom

Pau Mercadé Ruiz*, Francesco Ferri, Jens Peter Kofoed

Department of Civil Engineering, Aalborg University, Thomas Manns Vej 23, Aalborg 9220, Denmark

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ABSTRACT

In this article, a method to model wave diffraction and radiation by bodies of arbitrary shape over a variable bathymetry is developed. The effect of the bottom on the waves is modelled through the elliptic mild-slope equation, while the effect of the bodies through so-called diffraction transfer matrices. The numerical treatment proposed to solve the mild-slope equation is based on a finite-element discretisation of the fluid domain outside the bodies, while these are replaced by vertical surfaces over which analytical solutions exist. The solutions are then combined with diffraction transfer matrices to develop the required boundary conditions at the bodies. Full reflection and Sommerfeld's radiation condition at sea, where the water depth is assumed constant, are taken into account to fully determine the numerical solution. The method is further verified against exact solutions to the problem of tsunami response of a cylindrical island over a parabolic shoal, and the problem of wave diffraction by an array of truncated vertical cylinders over a flat bottom. Comparison between exact and present method wave amplitude solutions for the two problems show good agreement. Finally, the effects of a submarine plateau and a fully reflective coast on the wave diffraction and radiation by an array of surging barges are discussed.

1. Introduction

Most of human activities at sea are facilitated by the use of some type of marine structure, such as ships for ocean transportation, wind turbines for offshore wind energy exploitation, or wave energy converters for wave energy exploitation. The design of such structures nowadays involves using numerical tools that can model the interaction between the ocean waves and the structures. In the design of wave energy converters, linear wave models have been extensively used. This is because they are easier to solve than higher-order models and because the electric power produced by such devices stems largely from linear waves, i.e., waves with small amplitude compared to the wavelength and the water-depth; as these are the most frequent waves at sea. Under the action of such linear waves, the response of wave energy converters has been generally modelled as small amplitude oscillations compared to the wavelength and the water-depth. This is valid for a certain range of wave-frequencies; however, it becomes less accurate as the wave-frequency is closer to the resonance frequency of the devices.

There are three wave phenomena that are relevant when a linear wave intercepts a group of wave energy converters (solid bodies) over a variable bathymetry; these are known as wave refraction, diffraction and radiation. Wave refraction occurs due to varying water-depth, causing the wave to propagate at different velocities. Wave diffraction, on the other hand, occurs due to the presence of solid bodies as the water cannot flow through. Finally, wave radiation stands for the waves generated by solid bodies when moving about.

In the context of linearised potential flow and small body motions, the problem of combined wave refraction, diffraction and radiation may be formulated as a boundary value problem, which can be solved numerically, e.g., by the finite element method or the boundary element method. These two methods were proposed in Yue et al. (1978) and Matsui et al. (1987), respectively, in combination with an analytical representation of the wave-field outside a fictitious cylinder enclosing the bodies and the variable bathymetry, where the water-depth was assumed constant. In Belibassakis (2008) and Belibassakis et al. (2016), the boundary element method was also proposed, although unlike in Yue et al. (1978) and Matsui et al. (1987), the outer boundary was not required to be at constant water-depth.

Further, under the assumption of slowly varying water-depth, the boundary value problem reduces to the so-called mild-slope equation (Berkhoff, 1972). The mild-slope equation is desirable, when applicable, as it only needs to be solved in the horizontal plane. However, because of its two-dimensional nature, it can only handle wave

* Corresponding author.
E-mail address: pmr@civil.aau.dk (P. Mercadé Ruiz).

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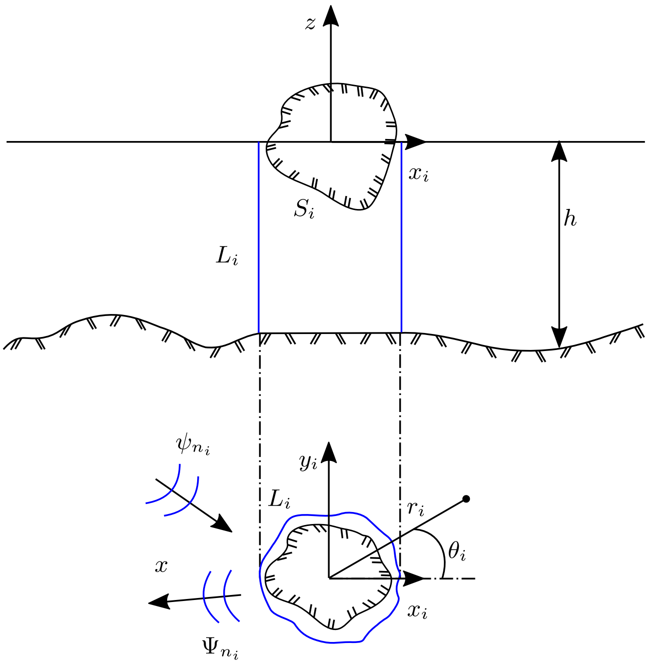

Fig. 1. Illustration of the problem variables for the modelling of combined refraction-diffraction. ψ_{n_i} represents the part of the waves entering the vertical surface L_i, whereas Ψ_{n_i} represents those leaving it.

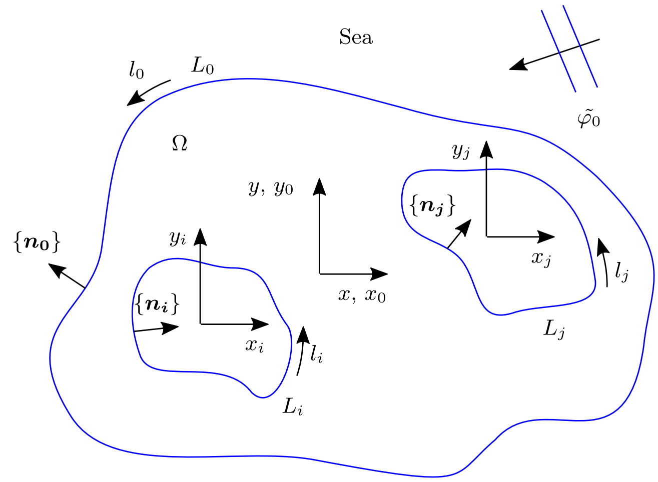

Fig. 2. Illustration of the problem variables for the modelling of combined refraction-diffraction in the two-dimensional finite domain $(x, y) \in \Omega$. $\widetilde{\varphi}_0$ represents the part of the waves entering the vertical surface L_0, which sets the boundary with the open sea.

diffraction by vertical walls and wave radiation due to horizontal displacements.

In order to extend the range of applicability of the elliptic mild-slope equation, the two-dimensional field equation, combined with the finite element method, and the full three-dimensional boundary value problem, applied only in the vicinity of the structure, were successfully coupled in Takagi and Naito (1994), Ohyama and Tsuchida (1994) and Cecioni and Bellotti (2016). The boundary element method was chosen in Takagi and Naito (1994) and Ohyama and Tsuchida (1994) to find a solution of the boundary value problem around the structure; continuity of solutions further led to the necessary boundary conditions for the coupling with the mild-slope equation model. On the other hand, a linear model based on Robin-type boundary conditions was chosen in Cecioni and Bellotti (2016) for the coupling instead, while the model coefficients were adjusted beforehand so as to fit the solution of the boundary value problem for the isolated structure. The accuracy of the matching between the mild-slope equation and the three-dimensional solutions in Takagi and Naito (1994), Ohyama and Tsuchida (1994) and Cecioni and Bellotti (2016) was though compromised by a minimum separation between the body and the interface where the matching was enforced.

The method presented in this paper aims to pose an alternative to those in Takagi and Naito (1994), Ohyama and Tsuchida (1994) and Cecioni and Bellotti (2016). This is done by adapting some of the main components of the so-called direct matrix method (Kagemoto and Yue, 1986) to the elliptic mild-slope equation. The direct matrix method have already been implemented into the DTOcean (Optimal Design Tools for Ocean Energy Arrays) software Optimal design tools for ocean energy arrays, for the modelling of wave diffraction and radiation by wave energy converter arrays; however, it is based on the assumption of constant water-depth. Therefore, the present method enables the modelling of wave energy converter arrays over a variable bathymetry from the already existing tools (Mercadé Ruiz et al., 2017) embedded in the DTOcean software.

Wave diffraction and radiation by bodies of arbitrary shape are thereby integrated with the elliptic mild-slope equation. However, here, an analytical representation of the diffracted (and radiated) wave-field near the body along with the single-body diffraction (and radiation) characteristics suggested in Kagemoto and Yue (1986) are used to produce the necessary boundary conditions for the coupling with the mild-slope equation. The present method is further combined with the techniques developed in McNatt et al. (2015), implemented into the DTOcean software, to facilitate the calculation of the single-body diffraction (and radiation) characteristics and the calculation of post-processed wave forces. In addition, by using these techniques, the accuracy of the matching between the mild-slope equation and the three-dimensional solutions is not compromised by a minimum separation between the body and the matching interface.

The work presented in this paper is structured in five sections. Section 2 provides the equations for the modelling of combined refraction-diffraction-radiation. Section 3 provides a numerical scheme for the solution of the problem of combined refraction-diffraction-radiation. Section 4 is devoted to the verification of the method by comparing the wave-field solved by the present method with that solved analytically for two different test problems. In addition, two more test problems are discussed in Section 4 which illustrate the applicability of the method. Finally, Section 5 discusses the adequacy of the choice of the method parameters and provides recommendations for the scalability of the results.

2. Method formulation

Consider a harmonic wave travelling towards a group of I three-dimensional bodies. The wave transformations caused by the presence of the bodies (diffraction), the motion of the bodies (radiation) and the variable bathymetry (refraction) are addressed in this section.

In the context of potential flow theory, the wave-field can be represented by the velocity potential Φ, whose gradient yields the fluid velocity vector. In addition, since only harmonic waves are considered, it seems convenient to represent the potential using complex notation, i.e., $\Phi = \Re(\phi\ \mathrm{e}^{-\mathrm{i}\omega t})$; where $\omega = 2\pi/T$ is the wave angular frequency, T is the wave-period, $\mathrm{i} = \sqrt{-1}$ is the imaginary unit, t is time, and ϕ is the complex amplitude of the potential.

The potential must then satisfy the conservation of mass equation at any point in the fluid domain, yielding the well-known Laplace's equation:

$$\Delta\phi + \frac{\partial^2\phi}{\partial z^2} = 0; \tag{1}$$

where $\Delta = \nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$, $\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right]$ and (x, y, z) are Cartesian coordinates.