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Statistical prediction of parametric roll using FORM

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ABSTRACT

Previous research has shown that the First Order Reliability Method (FORM) can be an efficient method for estimation of outcrossing rates and extreme value statistics for stationary stochastic processes. This is so also for bifurcation type of processes like parametric roll of ships. The present paper discusses this solution procedure with a focus on the computational efficiency of FORM as compared with Monte Carlo Simulation (MCS).

1. Introduction

Currently, extensive work is going on within the International Maritime Organization (IMO) in the development of Second Generation Intact Stability Criteria for ships. These completely revised rules include the possibility to account for the dynamics of ships using time-domain simulations of the roll motion under different operational conditions, considering different failure scenarios (pure loss of stability, parametric roll, dead ship, excessive acceleration and surf riding/broaching) and involve different levels of complexity and corresponding accuracy. Tompuri et al. (2015) discuss in details computational methods to be used in the Second Generation Intact Stability Criteria, focusing on level 1 and level 2 procedures for parametric roll, pure loss of stability and surf-riding/broaching. These methods are based on the analysis of the ship in regular waves with different wave height and thus do not directly provide extreme value statistics.

The rules might not only be used in the design phase, but also be needed under operation as *GM* limit curves cannot always be formulated using the new rules, e.g. IMO (2017). For the more detailed analyses in Level 3, and for operational guidelines/limitations, a direct account for the statistical properties of the ocean waves will be needed and so will effective statistical estimation procedures to cover the full operational profile of a vessel.

France et al. (2003) present an excellent and very thorough description of the physics in parametric roll; with discussions based on both numerical studies and model test results. Hence, the present paper will focus on an extreme value prediction procedure applicable as an extension to more deterministic formulations of parametric roll.

The First Order Reliability Method (FORM) is an efficient procedure for extreme value predictions for time-invariant stochastic processes, e.g. Der Kiureghian (2000), Jensen and Capul (2006), Jensen (2015), Jensen (2007) uses FORM for estimation of the probability of parametric roll. In the present paper the same one degree-of-freedom formulation of parametric roll is used, but two simple and effective optimization procedures, easy to implement in any time-domain code for FORM evaluations, are presented. Furthermore, some characteristic response behaviour in parametric roll is discussed. A recent study by Choi et al. (2017) gives a somewhat similar treatment of intact stability under dead ship conditions.

2. First order reliability procedures

The basic assumption for the application of the FORM method herein is that the response can be considered as a stationary time-domain process, depending solely on a load process in time t and space X, defined in terms of some deterministic quantities and a set of statistical independent and standard normal distributed variables $\underline{u} = \{u_i, \overline{u}_i; i = 1, 2, .., n\}$. For wave responses, the long-crested wave elevation process $H(X,t)$ is such a process, e.g. Jensen and Capul (2006):

$$H(X,t) = \sum_{i=1}^{n}(u_i c_i(X,t) + \overline{u}_i \overline{c}_i(X,t)) \tag{1}$$

where the deterministic coefficients are

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$$c_i(x,t) = \sigma_i \cos(\omega_i t - k_i X); \quad \bar{c}_i(x,t) = -\sigma_i \sin(\omega_i t - k_i X); \quad \sigma_i^2 = S(\omega_i) d\omega_i \tag{2}$$

Here ω_i and $k_i = \omega_i^2/g$ are the n discrete frequencies and wave numbers applied, respectively. Furthermore, g is the acceleration of gravity, $S(\omega)$ is the wave spectrum and, $d\omega_i$ is the increment between the discrete frequencies. Stochastic wind speed can also be modelled in a similar way, e.g. Choi et al. (2017).

For a set of $\underline{u}$, the wave elevation is used as input to a time-domain formulation for the response $\phi(t, \underline{u})$. Due to the assumption of a stationary stochastic process, the response at any point in time $t = t_0$ can be applied in the limit state function

$$G(\underline{u}) = \phi_0 - \phi(t_0, \underline{u}) = 0 \tag{3}$$

without changing the result for the probability of exceedance at a given threshold response level ϕ_0. The only restriction is that the point in time t_0 chosen must be so far away from the initial conditions that these do not influence the response. For parametric roll 300s was found in Jensen (2007) to be sufficient. For other wave responses 60s might be sufficient, e.g. Jensen (2015).

The main part of any FORM procedure is an optimization routine for determination of the point $\underline{u}^*$ on the limit state function $G(\underline{u}) = 0$ with the shortest distance from origin. The distance to this point is denoted the reliability index β. Two optimization routines: 1) A modified Hasofer-Lind procedure (MHL) and, 2) the Hasofer-Lind method supplemented with a circle and line search (CLS) are used here.

In the original Hasofer-Lind iteration procedure, Hasofer and Lind (1974), a new iteration point $\underline{u}_{k+1}$ is determined from the previous point $\underline{u}_k$ as

$$\underline{u}_{k+1} = \underline{a}_k = [\nabla G(\underline{u}_k)\underline{u}_k - G(\underline{u}_k)] \frac{\nabla G(\underline{u}_k)}{|\nabla G(\underline{u}_k)|^2} \tag{4}$$

where ∇ is the gradient operator and || the length of the vector. However, for the problems considered here, this procedure does not generally converge towards the design point $\underline{u}^*$. This lack of robustness of the Hasofer-Lind procedure is discussed in details in Liu and Der Kiureghian (1991) and several remedies are suggested.

In the Modified Hasofer-Lind method, Liu and Der Kiureghian (1991), the new iteration point $\underline{u}_{k+1}$ is determined from a line search along the line:

$$\underline{u} = \varsigma \underline{a}_k + (1 - \varsigma)\underline{u}_k \tag{5}$$

where $\underline{a}_k$ is given by Eq. (4). The scalar ς is determined by a simple stepping procedure until the merit function $m(\underline{u})$

$$m(\underline{u}) = \left[\underline{u} - \frac{\nabla G(\underline{u})\underline{u}}{|\nabla G(\underline{u})|^2} \nabla G(\underline{u})\right]^2 + c[G(\underline{u})]^2 \tag{6}$$

attains a minimum value, yielding

$$\underline{u}_{k+1} = \underline{u}\left\{ \underset{\varsigma}{m}(\underline{u}) = \min \right\} \tag{7}$$

The weight factor c can be taken in a wide range from 1000 to 10,000 without changing the convergence significantly for the present problems, where the response ϕ_0 in the limit state function, Eq. (3), is the roll angle in radians. This insensitivity of convergence rate with c is in agreement with the findings in Liu and Der Kiureghian (1991) considering very different examples. The range investigated for ς is $\varsigma \in]0, 2]$ with a step size of 0.025. The procedure is very easy to implement and convergence is found in all cases considered here. The procedure is, however, rather CPU expensive as it requires gradient calculations $\nabla G(\underline{u})$ for all values of ς used in Eq. (5).

An alternative is the Hasofer-Lind procedure supplemented with a circle and line search, Choi et al. (2017). Based on the previous iteration step $\underline{u}_k$ the new iteration point $\underline{u}_{k+1}$ is determined from first a circle search along the circle:

$$\underline{u} = \frac{|\underline{a}_k|}{|\varsigma \underline{a}_k + (1 - \varsigma)\underline{u}_k|} (\varsigma \underline{a}_k + (1 - \varsigma)\underline{u}_k) \tag{8}$$

where $\underline{a}_k$ is given by Eq. (4). The scalar ς is determined by a simple stepping procedure until the limit state function $G(\underline{u})$ attains a minimum value. The corresponding value of $\underline{u}$ is denoted $\tilde{\underline{u}}$. Thereafter a line search along the line $\underline{u} = \xi \tilde{\underline{u}}$ is performed. The scalar ξ is determined such that $G(\underline{u}) = 0$ yielding

$$\underline{u}_{k+1} = \xi \tilde{\underline{u}} \left\{ \underset{\xi}{G}\left(\xi \tilde{\underline{u}}\right) = 0 \right\} \tag{9}$$

A Newton-Raphson approach is applied based on previous values at iteration step *i* and *i-1*:

$$\xi_{i+1} = \xi_i - \frac{\xi_i - \xi_{i-1}}{G\left(\xi_i \tilde{\underline{u}}\right) - G\left(\xi_{i-1}\tilde{\underline{u}}\right) G\left(\xi_i \tilde{\underline{u}}\right)} \tag{10}$$

For the first step, $\xi_1 = 1 + 0.01G(\tilde{u})/G_e$ is found useful. Here, G_e is the user-defined convergence criterion for the limit state function. With the threshold angles ϕ_0 measured in radians, $G_e = 0.002$ has been found adequate.

The convergence property of this scheme is just as good as for the MHL procedure, in all cases considered, and the scheme provides a large reduction of CPU time although the number of iteration steps generally is larger. The reason is that gradient calculations are not needed during the circle–and-line search as opposite to the MHL method. With a large number of components in $\underline{u}$, say 100 as used later in the example, the CPU time is thereby reduced by a factor of three to five. Both procedures, however, converge for all cases tested to the same design point $\underline{u} = \underline{u}^*$.

The distance to the design point $\underline{u}^*$ is the reliability index β and from it extreme value predictions can easily be obtained, e.g. Jensen and Capul (2006),

$$P\left[\max_T \phi(t) > \phi_0\right] = 1 - \exp\left(-\nu_0 T \exp\left(-0.5\beta(\phi_0)^2\right)\right) \tag{11}$$

here T is the time period considered, e.g. 3 h, and ν_0 the mean zero-upcrossing rate, roughly equal to the roll natural frequency in Hz. It should, however, be noted that Eq. (11) does not account for possible grouping of the threshold angles. This is not investigated here, where the focus is on comparison between reliability indices from FORM and MSC, but the procedure suggested by Naess and Gaidai (2009) could for instance be implemented in the time domain simulations to account for the clustering effect. Thereby, Eq. (11) is replaced by a somewhat similar expression, Naess and Gaidai (2009). This is a topic for a current investigation.

3. Effective wave for GZ calculations

Parametric roll in head sea depends on the variation of the instantaneous GZ curve in waves. In principle the roll restoring moment can be calculated at each point in time, e.g. Vidic-Perunovic and Jensen (2009), but this is computationally expensive. Therefore it is often estimated by interpolation in predefined GZ curves derived from hydrostatic results with the ship 'resting' in regular waves with a wave length equal to the length L of the vessel. The wave height $h(t)$ and wave crest position $x_c(t)$ used in this interpolation are found by a least square approximation to the incident wave $H(X, t)$, Eqs. (1)–(2), cf. Jensen (2007):