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An *hp*-adaptive discontinuous Galerkin method for modelling snap loads in mooring cables

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ABSTRACT

This paper focuses on modelling snap loads in mooring cables. Snap loads are a known problem for the established oil and gas industry, and they pose a major challenge to robust mooring design for the growing industry of wave energy conversion. We present a discontinuous Galerkin formulation using a local Lax-Friedrich Riemann solver to capture snap loads in mooring cables with high accuracy. An $hp-$adaptive scheme is used to dynamically change the mesh size h and the polynomial order p, based on the local solution quality. We implement an error indicator and a shock identifier to capture shocks with slope-limited linear elements, while using high-order Legendre polynomials for smooth solution regions. The results show exponential error convergence of order $p+1/2$ for smooth solutions. Efficient and accurate computations of idealised shock waves in both linear and nonlinear materials were achieved using $hp-$adaptivity. Comparison with experimental data gives excellent results, including snap load propagation in a mooring chain. Application on a wave energy device using coupled simulations highlights the importance of the touch-down region in catenary moorings. We conclude that the formulation is able to handle snap loads with good accuracy, with implications for both maximum peak load and fatigue load estimates of mooring cables.

1. Introduction

Snap loads are an important factor in the structural design of marine cable installations. For example, they need to be considered during marine lifting operations (Bauduin et al., 2015) and they are known to cause mooring line failure for floating oil production installations (Safetec, 2013). The snap phenomenon can result in high peak loads and increased fatigue damage of cable installations. For the emerging field of wave energy converters (WECs) that put larger demands on the mooring system design and functionality (Johanning et al., 2007; Fitzgerald, 2009), snap loads are potentially an even larger hazard to the design. Reports show that snap loads can cause great damage in both experiments and field tests of WECs (Hann et al., 2015; Thies et al., 2012; Harnois, 2014; Savin et al., 2012). However, firm conclusions on snap load occurrence and the resulting amplitude is difficult to reach from measurements only (Harnois, 2014). It is therefore important that numerical methods used for cable dynamics are able to handle snap loading events properly.

Snap loads are characterised by a discontinuity in tension magnitude that propagates along the cable (Dhanak and Nikolaos, 2016). There are three main mechanisms by which snap loads are generated in mooring cables. First, there is the shock wave build up due to nonlinear material response. Tjavaras (1996) studied these shock conditions in highly extensible fibre ropes using the method of characteristics and finite differences. He showed how shocks form in fibre ropes with exponential strain-tension behaviour. A second snap load generation mechanism arises from sea-bed contact, predominantly in catenary slack moorings. Triantafyllou et al. (1985) has showed that a snap is generated when the touch-down point velocity of a chain exceeds the wave-speed in the transverse direction of the cable. This was later observed in experiments by Ref. Gobat and Grosenbaugh (2001) and computed with good results by Ref. Gobat (2000) using finite differences and adaptive time-stepping. The third and most common snap load is however associated with the cable slack condition. The snap load amplitude is in this case dependent on the material stiffness and the local strain rate of the cable at the instant it re-enters the tensioned regime (Hennessey et al., 2005). The experiments of Fylling and Wold (1979) investigated snap loads of this type. They have been numerically studied by several authors, e.g. Shin (1991) using a clipping model that showed that the snap amplitude decreased with increasing free-falling velocity of the cable. Also Vassalos and Kourouklis (1998) used the lumped mass method as described in

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Ref. Huang (1994) to compare with said experiments. Good results were obtained for cases with smooth dynamic response, but errors up to 30% were noted for cases with snap loads. We note that in the case of cable slack at the contact point, these definitions overlap and the governing mechanism for the snap is a mixture of the second and third types of snap load generation.

There are a multitude of numerical cable formulations and models; see e.g. Ref. Spak et al. for a good review and Brown and Mavrakos (1999) for a comparative benchmark test between different methods. A common cable discretisation technique is to use discrete lumped masses. This was originally described by Ref. Walton and Polachek (1959), and is frequently used today (Orcina Inc, 2012; ANSYS Inc, 2013). In early work, a number of investigations were also made using finite differences (Tjavaras, 1996; Gobat and Grosenbaugh, 2001; Ablow and Schechter, 1983; Mavrakos et al., 1996). Linear finite element formulations include the work of Aamo and Fossen (2000), and commercial solvers such as DeepC (DNV GL, 2014). A Galerkin method based on cubic splines was introduced by Ref. Buckham et al. (2004) as a starting point to higher order modelling of the cables. Of particular importance to this work is the paper of Montano et al. (2007) who formulated a mixed high-order finite element model for cables. The position and velocity of the cable were modelled using continuous Galerkin finite elements of high order, but the tension was an auxiliary discontinuous Lagrangian multiplier constraint. Under the assumption of negligible bending stiffness, they showed good results for very stiff and inextensible cables. However, to propagate snap loads we need to resolve the time-scales of longitudinal waves of tension. This was the aim of our previous study, where we developed a local discontinuous Galerkin (LDG) method for mooring cables (Palm et al., 2013). The LDG formulation required stabilisation penalty terms as expected (Cockburn and Shu, 2001), but showed good results in convergence and validation tests. However, a constant choice of fluxes made snap load capturing difficult, showing a need for a more sophisticated numerical scheme.

The governing equation of mooring cable dynamics is hyperbolic (Tjavaras, 1996; Montano et al., 2007), and shock waves in hyperbolic conservation laws is a well studied topic. The theorems of Lax and Wendroff (1960), and of Hou and Le Floch state that any converging solution of a shock in a hyperbolic equation will only converge to the correct (and unique) solution if the problem is formulated in conservative form. Discrete representations of shocks are also subject to Ref. Godunov (1959) theorem stating that all constant flux schemes of orders greater than one will produce non-physical extrema (over/undershoots) in the presence of discontinuities. The total variation diminishing (TVD) family of flux-limiters (see e.g. Ref. Sweby (1984)) have been developed to remedy the accuracy for second order finite volume simulations.

Shocks can be modelled accurately using discontinuous Galerkin (DG) methods in conservative form. The DG method is essentially a finite volume scheme with each cell approximated using finite elements. The elements are connected via numerical fluxes, like in the finite volume method. Shape functions of arbitrary polynomial order can be used to achieve exponential convergence for smooth solutions (Karniadakis and Sherwin, 2003), enabling engineering accuracy with only a few elements. However, in the presence of shocks, the estimated amplitude will be affected by overshoots and undershoots around the shock front of the solution (Toro, 2001). There are many approaches to capture shocks, where the main is through limiting the flux (or slope) of the solution as in finite volume schemes, see e.g. Ref. Sweby (1984). Among other techniques we note the artificial viscosity for sub-cell shocks by Ref. Persson and Peraire (2006) and the moment limiters for high order meshes (Krivodonova, 2007). These measures have in several studies been combined with mesh adaptivity in element density (h) and/or polynomial order of the expansion basis (p) (Berger and Colella, 1989; Bey et al., 1996; Eskilsson, 2011), as well as with shock detection schemes (Bernard, 2008; Krivodonova et al., 2004).

We present a high-order discontinuous Galerkin (DG) method for cable dynamics with the purpose of capturing and resolving snap loads. The problem is formulated in conservative form, including an approximative Riemann solver based on the local Lax-Friedrich flux. Further, an $hp-$adaptive strategy based on the tension magnitude is applied. The $hp-$adaptivity aims to utilise the desirable accuracy of high-order elements in smooth regions, while returning to slope limited linear elements around the discontinuities, to resolve the shocks. Computational results are compared with analytic results for three idealised test cases. Further, we compare computational results with experimental data from a mooring chain subjected to prescribed end-point motion.

The paper is organised as follows. First we present the governing equations, recasted in conservative form, and the physical assumptions made in the derivation (Section 2). This is followed by an eigenvalue analysis of the model system (Section 3). Section 4 describes the details of the numerical model implementation, with the $hp-$adaptive strategy presented in the following Section 5. Computational examples are then presented in Section 6 and the paper ends with concluding remarks in Section 7.

2. Governing equations

For a cable of length L_c, we use the unstretched cable coordinate $s \in [0, L_c]$ to express the global coordinate position vector of the cable as $r = [r_1(s), r_2(s), r_3(s)]^{\mathrm{T}}$. Under the assumption of negligible bending stiffness, the equation of motion becomes

$$\gamma_0 \ddot{r} = \frac{\partial}{\partial s}(T\hat{t}) + f \ , \tag{1}$$

$$\hat{t} = \frac{\partial r}{\partial s}\left|\frac{\partial r}{\partial s}\right|^{-1} \ , \tag{2}$$

where γ_0 is the cable mass per unit length, T is the cable tension force magnitude, $\hat{t}$ is the tangential unit vector of the cable and f represents all external forces. For notation we use $\dot{x} = \frac{\partial x}{\partial t}$ to indicate time derivatives and $|x| = \sqrt{x_i x_i}$ to denote the L_2 - norm of a vector quantity x, Vector components are denoted by their index as $x_i, \ i \in [1, 2, 3]$, and summation over repeated indices is implied.

Written as a first order system in terms of the cable position r, its spatial derivative $q = \frac{\partial r}{\partial s}$ and its momentum density $\nu = \dot{r}\gamma_0$, eq. (1) becomes

$$\dot{r} = \frac{\nu}{\gamma_0} \ , \tag{3}$$

$$\dot{q} = \frac{\partial}{\partial s}\left(\frac{\nu}{\gamma_0}\right), \tag{4}$$

$$\dot{\nu} = \frac{\partial}{\partial s}(T\hat{t}) + f, \tag{5}$$

where we have assumed that the cable mass is constant in time. In terms of a state vector $u = [r, q, \nu]^{\mathrm{T}}$ the conservative form of the problem is written as

$$\dot{u} = \frac{\partial F(u)}{\partial s} + Q(u) \ , \tag{6}$$

with a flux function

$$F(u) = \left[\varnothing, \frac{\nu}{\gamma_0}, T\hat{t}\right]^{\mathrm{T}} \ , \tag{7}$$

and a non-linear source term

$$Q(u) = \left[\frac{\nu}{\gamma_0}, \varnothing, f\right]^{\mathrm{T}} \ . \tag{8}$$