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Square Error Method for threshold estimation in extreme value analysis of wave heights

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ABSTRACT

One of the methods used to model the probability distribution of extreme values is the peaks over threshold method. It requires an appropriate estimation of the threshold above which exceedances are considered to be a sample from a generalized Pareto distribution. Several methods have been proposed to determine the optimal threshold. In this paper, we assess the applicability of four methods developed for hydrological, coastal engineering, financial and stock market purposes, to the case of wave heights from extreme sea conditions due to storms, where significantly less amount of data may be available. Based on the results of this assessment, an improved square error method is formulated, which further allows evaluating the uncertainty in threshold estimation, as well as the uncertainty in the estimation of wave heights for given return periods. The improved method is applied using significant wave height data from hindcast of tropical and extra-tropical storms in the Gulf of Mexico. Results illustrate the capabilities of the proposed method.

1. Introduction

The characterization of extreme values of meteorological and oceanographical (metocean) variables is critical for the design and operation of offshore structures and production systems. The estimation of the probability distributions of extreme values of metocean variables is generally based on the Peaks Over Threshold (POT) method, provided that exceedances over a sufficiently high threshold approach the Generalized Pareto Distribution (GPD); see e.g. Ferreira and Guedes (1998), Pickands (1975), Balkema and de Haan (1974), Salvadori et al. (2007), Reiss and Thomas (2007) and Falk et al. (2004). Applying the POT method requires choosing an appropriate threshold, which is a process that implies a balance between bias and variance. Low thresholds result in samples with many observations, including data that are not true extremes, and hence, the distributions estimates have low variance but large bias. On the contrary, high level thresholds provide fewer observations and thus estimates have a greater variance but less bias. For a review on recent advances and traditional techniques for threshold estimation in extreme value applications please refer to Scarrott and MacDonald (2012). In their review, available methods for threshold estimation are grouped into graphical diagnostics, empirical methods or "rules of thumb", probabilistic based techniques, computational approaches and mixture models.

An approach that has been used for threshold selection is to examine the behavior of two parameters which in theory remain constant under variations of threshold u: the shape parameter ξ of the GPD and a modified scale parameter $\tau = \sigma_u - \xi u$, where σ_u is the scale parameter of the GPD (Mallor et al., 2009; Thompson et al., 2009). The appropriate threshold is taken as the value of u above which ξ and τ are stable. Della-Marta et al. (2009) studied the asymptotic independence of the shape and the modified scale parameters and applied it to determine appropriate thresholds for extreme value analysis of wind data. Tanaka and Takara (2002) concluded that such approach performed very well to select the appropriate threshold for the characterization of extreme floods.

Methods based on graphical diagnostics have also been employed. They include the mean excess plot, the threshold stability plot, and the Hill plot. For instance, in financing, the Hill plot has been used for threshold selection in modeling tail distributions of returns and losses (Andreev et al., 2010). The interpretation of graphical methods demands substantial expertise and involves subjective criteria of the analyst. More sophisticated methods have been developed, such as the use of mixture methods. They combine GPD distribution models for the extreme data above the threshold with appropriate models for the bulk distribution of the non-extreme data. In these approaches the threshold is a parameter to be estimated and in most cases the uncertainty in such estimation can be accounted for (see e.g. Frigessi et al., 2003, Tancredi et al., 2006). Variations of mixture methods may consider parametric or non-parametric models for the bulk distribution. These methods may be computationally intensive and may not be easy to

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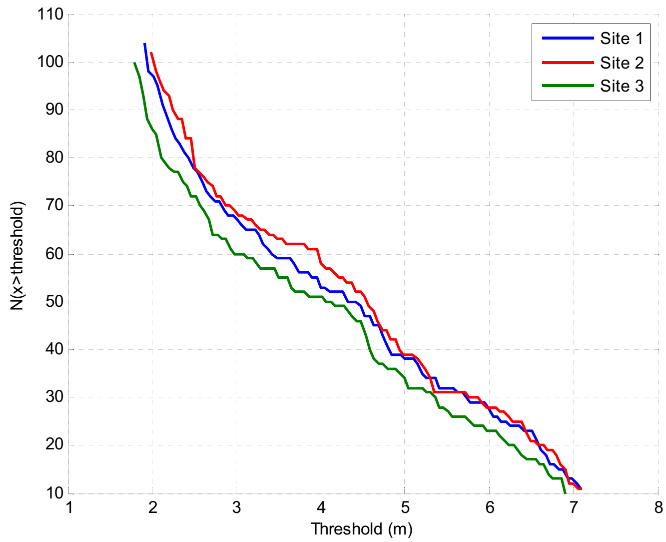

Fig. 1. Variation of the number of data with threshold.

implement for practical engineering applications.

In the field of financial mathematics, a quantitative method has been proposed to estimate thresholds based on the Hill estimator (Zhou et al., 2007). It defines the optimal threshold as that where the deviation of the Hill estimator from a linear approximation is the largest. The method was applied to analyze a sample of copper trading prices containing 2873 data points. It is simple to use and easy to implement in a computer program, which makes it attractive from a practical point of view. Thompson et al. (2009) have developed an automated threshold selection method. The procedure consists in finding, among a range of candidate threshold values, the one for which a sample sequence of a parameter, defined in terms of the GPD parameter estimates, is consistent with a zero mean normal distribution. The method was applied to a large data set of daily rainfall from a 48 year period and to coastal wave data with 10,000 sample observations. Zoglat et al. (2013) have proposed an algorithm based on the work of Beirlant et al. (1996) where the optimum threshold is selected to minimize a mean square error of the tail index Hill estimator. The Square Error Method (SEM) proposed by Zoglat et al. (2013) is defined in terms of observed and simulated quantiles from GPDs whose parameters are estimated based on the exceedances over each of the candidate thresholds. The optimal threshold is the one that minimizes the square error. The method was applied to a sample of 5222 observations of a stock market index. Deidda (2010) has proposed the multiple threshold method (MTM) which aims at estimating a threshold-invariant distribution function that assures a perfect overlap with GPDs fitted to the exceedances over any threshold larger than the optimum one. Estimation of an optimal threshold is not the purpose of the method; however, visual inspection of the invariant distribution helps to infer the optimal threshold. The method was applied using 217 time series of more than 40 complete years of daily rainfall records.

In this work, we assess the applicability of the Hill estimator method, the automated threshold selection method, and the SEM, which provide quantitative estimations of the threshold and are not highly computational intensive, to the case of metocean data for extreme sea conditions due to storms, where significantly less amount of data may be available. The MTM is also considered for its different approach and relative computational ease. Based on the assessment of the performance of these four methods, advantages are found in using the SEM and some necessary improvements of this method are identified. An improved SEM is then formulated and applied to significant wave height data from hindcast of tropical and extra-tropical storms in the Gulf of Mexico. Next, results of threshold estimation, and of assessment of uncertainty in threshold estimation

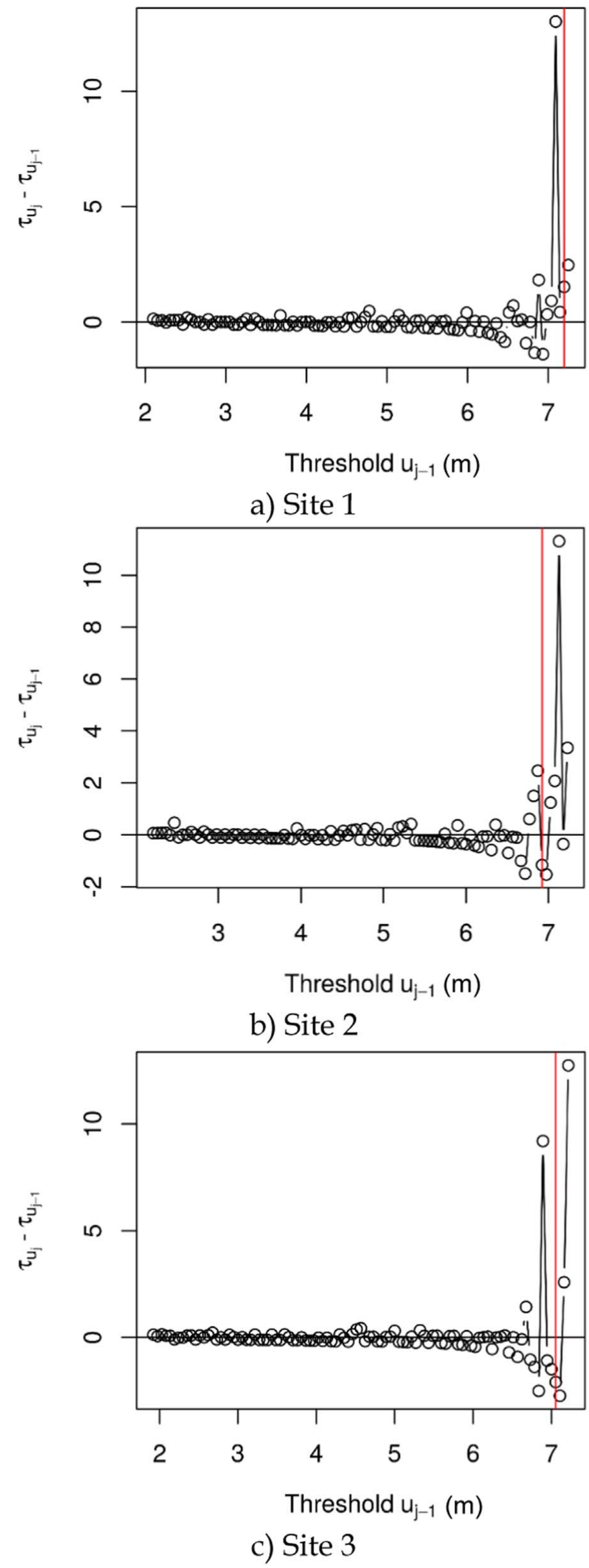

Fig. 2. Sequence $\tau_{u_j}-\tau_{u_{j-1}}$ vs threshold u_{j-1}; optimal threshold depicted by vertical line.

and return values, are given, showing the appropriateness of the method. The main features and advantages of the improved method are summarized in the conclusions.

2. Performance of threshold estimation methods

In the following, the steps of the selected methods are outlined