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Compensating of added mass terms in dynamically positioned surface vehicles: A continuous robust control approach

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ABSTRACT

In this work, we provide a tracking controller formulation for dynamically positioned surface vessels with an asymmetric added mass terms that affects the overall system dynamics at the acceleration level. Specifically a novel continuous robust controller is proposed for surface vessels that in addition to unstructured uncertainties in its dynamics, contains added mass effects in its inertia matrix. The proposed controller compensates the overall system uncertainties while ensuring asymptotic tracking by utilizing the knowledge of the leading principal minors of the input gain matrix. Stability of the closed–loop system and asymptotic convergence are proven via Lyapunov based approaches. Simulation studies are also presented to illustrate the viability of the proposed method.

1. Introduction and system model

The mathematical model for a dynamically positioned fully actuated 3 degree of freedom (dof) surface vessel is commonly represented by Fossen (2011,1994,2002), Skjetne et al. (2004), Ihle et al. (2006).

$$M_s\dot{v} + C_s v + D_s v = \tau \tag{1}$$

$$\dot{x} = Rv \tag{2}$$

where $x(t) \triangleq [x_p,\ y_p,\ \psi]^T \in \mathbb{R}^3$ is the position vector that contains translational positions $x_p(t),\ y_p(t) \in \mathbb{R}$ in X– and Y– directions, respectively, and the yaw angle of the ship $\psi(t) \in \mathbb{R}$, $v(t) = [u,\ v,\ \dot{\psi}]^T \in \mathbb{R}^3$ is the body–fixed linear and angular velocity vector. Also in (1), $M_s(\psi)$, $C_s(v, v_r)$, $D_s(v, v_r) \in \mathbb{R}^{3\times3}$ represent inertia matrix, centripetal and Coriolis forces, hydrodynamic damping terms, respectively, $v_r(t) \in \mathbb{R}^3$ is the relative velocity between the fluids and the vessel, and the control input vector is represented by $\tau(t) \in \mathbb{R}^3$. In (2), $R(\psi) \in SO(3)$ is the rotation matrix that has the form

$$R(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

While the mathematical model in (1) and (2) are utilized in almost all past work, as detailed in Fossen (1994,2002), Skjetne et al. (2004), and Fossen and Strand (1999), during the cruise, the motion of the surface vessel effects all the flow, resulting in vibrations with different amplitudes to occur on various parts of the flow. It is important to note that motion in one direction cause forces not only in the same direction but also in other directions (Newman, 1977; Lewis, 1989). This situation results as pressure effects and moments acting on different parts of the surface vessels and submarines which causes additional force and thus has an influence on the acceleration of surface vessels and submarines. For precise control design, this effect, referred as the added mass, is required to be represented in the dynamic model. There are different conventions (Skjetne et al., 2004; Fossen and Strand, 1999; Kim et al., 2007) on how to represent the added mass effects in the dynamic model. To name a few: In Fossen and Strand (1999), after using inertial velocity as the velocity state, the added mass effects are represented inside the system's inertia matrix. Following the convention given in Fossen and Strand (1999), in this work, the added mass terms are considered to be affecting the system dynamics at the acceleration level (i.e., inertial velocity is chosen as the velocity state). As a result, the inertia matrix of the surface vessel in (1) is obtained as (Fossen, 1994)

$$M_s = M_{RB} + M_A \tag{4}$$

where $M_{RB}(\psi) \in \mathbb{R}^{3\times3}$ represents the positive definite, symmetric rigid body inertia matrix and $M_A(\psi) \in \mathbb{R}^{3\times3}$ represents the added mass inertia matrix. The entries of added mass inertia matrix represented by $M_{A_{ij}}$ denote the mass associated with a force on the body in the ith direction due to a unit acceleration in the jth direction (Techet, 2015).

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For a detailed representation of the mathematical model reader can refer to Lee et al. (2008).

As noted in Fossen (1994) the inertia matrix due to added mass is not necessarily symmetric. After summed with the symmetric $M_{RB}(\psi)$, the overall inertia matrix $M_s(\psi)$ of the system loses its symmetry. The asymmetric terms in the inertia matrix, when not appropriately dealt with, may result in degradation of the controller performance, and even instability (Lee et al., 2008, 2008). Therefore the main challenge of added mass effects are due to its asymmetric nature. From a control design perspective, the symmetric nature of the inertia matrix is extremely useful especially when constructing quadratic terms in the Lyapunov candidate function.

To our best knowledge, there are only a few control design works that considered asymmetric added mass in the inertia matrix. In Do and Pan (2005), Do and Pan considered the case where the symmetry of the inertia matrix was removed for underactuated surface vehicles. Robust and adaptive type controllers were proposed for the fully actuated surface vehicles in Lee et al. (2008) and Lee et al. (2008), respectively. The aforementioned controllers were designed based on Lyapunov–type analysis methods, and were able to achieve only the ultimate boundedness of the tracking error signals.

In this work, a surface vessel having asymmetric inertia matrix is considered and output tracking control is aimed. The asymmetry of the inertia matrix is to be dealt with a matrix decomposition which while proposing a solution to the asymmetric inertia matrix by introducing a symmetric inertia–like matrix causes the control input to be pre–multiplied first with an uncertain unity upper triangular matrix and then with a known diagonal matrix. This is then addressed via the control design and accompanying stability analysis where first boundedness of all the closed–loop signals are demonstrated. Then via the use of two lemmas semi–global asymptotic stability of the tracking error is proven. Numerical simulations are then performed to demonstrate the viability of the proposed robust control strategy.

2. Open–loop error system development

In an attempt to obtain a compact representation of the mathematical model of the surface vessel in (1) and (2), the time derivative of (2) is taken

$$\ddot{x} = \dot{R}v + R\dot{v} \tag{5}$$

which includes the time derivative of the rotation matrix that can be obtained as

$$\dot{R} = RS_3 \tag{6}$$

with $S_3(\dot{\psi}) \in \mathbb{R}^{3\times3}$ being a skew–symmetric matrix defined as

$$S_3 \triangleq \dot{\psi}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{7}$$

After substituting (1) and (6) into (5), it is easy to obtain

$$\ddot{x} = RM_s^{-1}\tau - R[M_s^{-1}(C_s + D_s) - S_3]v. \tag{8}$$

In order to ease the presentation of the subsequent development, the right–hand side of (8) can be rewritten as

$$\ddot{x} = h + G\tau \tag{9}$$

where $h(x, \dot{x}) \in \mathbb{R}^3$ and $G(x, \dot{x}) \in \mathbb{R}^{3\times3}$ are defined as

$$h \triangleq -R[M_s^{-1}(C_s + D_s) - S_3]v \tag{10}$$

$$G \triangleq RM_s^{-1}. \tag{11}$$

It is assumed that $G(x, \dot{x})$ being a real matrix with non–zero leading principle minors by utilizing the assumption that $M_s(\psi)$ has full rank. This allows the utilization of the matrix decomposition in Costa et al. (2003), Morse (1993) to yield

$$G = SDU \tag{12}$$

where $S(x, \dot{x})$, D and $U(x, \dot{x}) \in \mathbb{R}^{3\times3}$ denote a symmetric positive definite matrix, a diagonal matrix with entries being ± 1, and a unity upper triangular matrix, respectively. When the above matrix decomposition is applied to the simulation model taken from Skjetne et al. (2004), D came out to be an identity matrix. Despite this, the derivations will be made for the general case where D is assumed to be available for the control design (see Chen et al., 2008 for a similar type of assumption).

From (9), via utilizing the assumption that the leading principal minors of G are non–zero, it is easy to obtain

$$\tau = G^{-1}(\ddot{x} - h). \tag{13}$$

After taking the time derivative of (9), substituting (12) and (13), and then performing straightforward mathematical manipulations yield

$$\dddot{x} = \varphi + SDU\dot{\tau} \tag{14}$$

where $\varphi(x, \dot{x}, \ddot{x}) \in \mathbb{R}^3$ is an auxiliary vector that is defined as

$$\varphi \triangleq \dot{h} + \dot{G}G^{-1}(\ddot{x} - h). \tag{15}$$

At this point, $M(x, \dot{x}) \in \mathbb{R}^{3\times3}$ is defined as the inverse of $S(x, \dot{x})$. Since $S(x, \dot{x})$ obtained from the matrix decomposition is symmetric and positive definite, so is $M(x, \dot{x})$. Furthermore, $M(x, \dot{x})$ satisfies the following inequalities

$$\underline{m}\ \|\chi\|^2 \le \chi^T M(x, \dot{x})\chi \le \overline{m}(x, \dot{x})\ \|\chi\|^2 \ \forall\ \chi \in \mathbb{R}^{3\times1} \tag{16}$$

where $\underline{m} \in \mathbb{R}$ and $\overline{m}(x, \dot{x}) \in \mathbb{R}$ represent a positive bounding constant and a positive non–decreasing function, respectively.

Pre–multiplying both sides of (14) with $M(x, \dot{x})$ yields

$$M\dddot{x} = f + DU\dot{\tau} \tag{17}$$

where $f(x, \dot{x}, \ddot{x}) \triangleq M\varphi \in \mathbb{R}^3$.

Ensuring that the translational positions and the yaw angle would track a given reference trajectory while, at the same time, ensuring the boundedness of all the signals under the closed–loop operation consists of our main control objectives. The control design is based on availability of $x(t)$ and $\dot{x}(t)$ (i.e., full–state feedback).

In order to quantify the tracking control objective, the output tracking error, $e_1(t) \in \mathbb{R}^3$, is defined as the difference between the reference trajectory and the position of the surface vessel as

$$e_1 \triangleq x_d - x \tag{18}$$

where $x_d(t) \in \mathbb{R}^3$ is the reference trajectory chosen smooth enough in the sense that

$$x_d(t) \in C^3 \quad \text{and} \quad x_d^{(i)}(t) \in \mathcal{L}_\infty, \quad i = 0, 1, 2, 3. \tag{19}$$

In order to eliminate the higher order terms from our stability analysis (i.e., only first order time derivatives to appear in the time derivative of the Lyapunov function), an auxiliary error signal, denoted by $e_2(t) \in \mathbb{R}^3$, and a filtered error term, denoted by $r(t) \in \mathbb{R}^3$, are defined as follows

$$e_2 \triangleq \dot{e}_1 + e_1 \tag{20}$$

$$r \triangleq \dot{e}_2 + \alpha e_2 \tag{21}$$

where $\alpha \in \mathbb{R}^{3\times3}$ is a positive definite, diagonal, constant gain matrix. After premultiplying the time derivative of (21) with M, the following expression can be obtained

$$M\dot{r} = M(\dddot{x}_d + \ddot{e}_1 + \alpha\dot{e}_2) - f - DU\dot{\tau} \tag{22}$$

where (17), (18) and (20) were utilized. To obtain a compact form for the right–hand side of (22), we define an auxiliary function $N(x, \dot{x}, \ddot{x}, x_d, \dot{x}_d, \ddot{x}_d, \dddot{x}_d, t) \in \mathbb{R}^3$ as follows

$$N \triangleq M(\dddot{x}_d + \ddot{e}_1 + \alpha\dot{e}_2) - f + e_2 + \frac{1}{2}\dot{M}r. \tag{23}$$