Ocean Engineering 138 (2017) 1–8

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Numerical modeling of ice-water system response based on Rankine source method and finite difference method

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ARTICLE INFO

Keywords:
ACV
Ice-breaking
Displacement response
Rankine source method
FDM

ABSTRACT

Based on the thin viscoelastic plate assumption and the potential flow theory, the unified mathematical model was derived for the ice-water vibration problem induced by air cushion vehicle (ACV) sailing on different media (pure ice sheet, pure water, broken ice, semi-water-semi-ice, etc.). In order to solve this model, a hybrid algorithm combining Rankine source method (RSM) and finite difference method (FDM) was proposed. The displacement response and stress distribution as well as ice-breaking effect of ice sheet can be obtained by using this numerical method. Besides, a comparison with the theoretical results or experiment results is made, and a good agreement is achieved between the calculation results and the existing research results. Finally, taking the pure ice surface and semi-water-semi-ice surface as an example, some calculations are made according to the typical physical parameters of ice sheet. Our calculations shows that the mathematical model and numerical method can not only calculate the ice displacement response and its distribution characteristics of the internal stress as well as ice-breaking effect, but also can capture the critical speed of ACV.

1. Introduction

It was observed that ACV could be used to rupture ice by taking advantage of the fact that moving load can generate waves in ice-water system. Under a certain speed (critical speed), the displacement response of ice was the largest. For the research of displacement response of ice sheet to the moving load in polar environment, the book by Squire et al. (1996) described in detail the theory and experiments prior to 1996. In the early days, the main purpose to carry out these studies was how to transform the ice cover areas into roads or runways. Therefore, these studies focuses on the load level and critical speed which were the main factors affecting vehicle safety operation on the ice sheet.

In order to exploit the moving load to break the ice sheet, some studies have been carried out in recent years. Kozin and Pogorelova (2003, 2007) studied the resistance and deformation characteristics of ice sheet when the ACV is moving on ice or broken ice based on the asymptotic Fourier analysis. Miles and Sneyd (2003) studied the two-dimensional response of ice sheet to an accelerating load base on the Fourier-Laplace transform method. Besides, ice displacement response to the pulse load (sinusoidal load and triangular pulse load) was also obtained by using the integral transform method (Hu et al., 2012, 2014). In the meanwhile, Zhang et al. (2016) derived the approximate formula for the critical speed in shallow water based on the theory of Kheysin (1963) and Nevel (1970). Părău and Dias (2002) and Bonnefoy et al. (2009) adopted dynamical systems theory and higher-order spectral (HOS) method respectively to study the response of the ice sheet, the calculated result of the ice deformation at the critical speed was finite. In order to better understand the development of ice-breaking mode, Lu et al. (2012) used the dynamic simulation software Ls-dyna to simulate the ice-breaking process and the ice-breaking mechanism of ACV was analyzed according to the simulation results, in the mean while, the practical operation mode of ACV was also discussed. Pogorelova and Kozin (2014) employed asymptotic analysis method and integral transform method to study the unsteady motion of a line load in a pool whose depth changed in the direction of load motion. In the experiment study aspect, it is well known that Takizawa's (1985) experiment and Squire's (1985) experiment are the most famous. In recent years, Kozin and Pogorelova (2008) carried out a submarine model test by using viscoelastic membrane as the ice-like material. Besides, according to the similarity theory, Zhang et al. (2014) established a ship-pool test system for the deformation of ice-like material measurement, and a series of experiments was carried out in towing channel with different water depths.

In previous researches, the integral transformation was the most common method to solve this problem (calculating the ice displacement response and wave resistance), but this method is convenient only under the simple boundary conditions (infinite ice sheet, uniform

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http://dx.doi.org/10.1016/j.oceaneng.2017.04.011
Received 1 December 2016; Received in revised form 3 March 2017; Accepted 9 April 2017
0029-8018/

water depth, etc.). While in this paper, we first construct a hybrid algorithm combining RSM and FDM to solve the ice-water vibration problem induced by ACV sailing on different media (pure ice sheet, pure water, broken ice, semi-water-semi-ice, etc.); Secondly, this method can not only get the results what the integral transformation method can, but also can make a preliminary analysis of ice rupture according to a certain ice-breaking criterion; Finally, due to the flexibility of numerical methods dealing with the boundary conditions, we hope to solve the ice-water vibration problem induced by ACV under complex boundary conditions (solid wall, non-uniform water depth, curved channel) in further research, and it is of great significance for offshore icebreaking and river icebreaking.

2. Mathematical model

In order to establish a unified ice-water vibration model, we assume a semi-infinite ice sheet of thickness h and density ρ_1 floating on water of uniform depth H and density ρ_2, the other half is a free surface. The boundary line between free surface and ice surface is a straight line, and ACV moves in a straight line parallel to this boundary with constant speed U. We establish a coordinate system that moves with the ACV, and the ACV moves in the positive direction of the x-axis, we take o-xyz to coincide with the undisturbed water surface, so that the upper undisturbed water surface is z=0 and the water bottom z=−H, specifically, as shown in Fig. 1.

In this coordinate system, the flow and ice deformation become steady-state problems. We assume the ice sheet can be treated as an isotropic, homogeneous, viscoelastic thin plate of uniform thickness, and the fluid motion beneath the ice is irrotational and incompressible and can be described by a velocity potential Φ which composed of flow velocity potential $-Ux$ and disturbance potential $\phi(x,y,z)$, so that the governing equation (Squire et al., 1996) of ice sheet or free surface can be written in a unified form:

$$-D(1-\tau U\partial/\partial x)\nabla^4 w+(p_W-p_A)-\rho_1 gh=\rho_1 hU^2\partial^2 w/\partial x^2 \tag{1}$$

where w is the vertical deflection of ice sheet or water surface; $D=Eh^3/[12(1-\mu^2)]$ is the modulus of rigidity which depends on Young's modulus E and Poisson's ration μ; g is the gravitational acceleration; τ is the relaxation time of ice sheet (refer to Kozin and Pogorelova, 2003); $\nabla^4=(\partial^2/\partial x^2+\partial^2/\partial y^2)$; p_A is the air cushion pressure on free surface or ice surface; p_w is the water pressure at the interface of ice and water.

In the water field, the disturbance potential satisfies the Laplace's equation

$$\partial^2\phi/\partial x^2+\partial^2\phi/\partial y^2+\partial^2\phi/\partial z^2=0 \quad (-\infty<x<\infty,-\infty<y<\infty,-H<z<0) \tag{2}$$

At the bottom of water body, the no-normal-flow condition is

$$\partial\phi/\partial z=0 \quad (-\infty<x<\infty,-\infty<y<\infty,z=-H) \tag{3}$$

On the free surface, the dynamics conditions is

$$\rho_2 gw-\rho_2 U\partial\phi/\partial x=-p_{Ae} \tag{4}$$

where p_{Ae}=p_A−$p_{A\infty}$ is the relative pressure on the free surface, $p_{A\infty}$ is the pressure on the free surface at infinity.

At the ice-water interface, linearized kinematic condition is

$$\partial\phi/\partial z=-U\partial w/\partial x \tag{5}$$

At the ice-water interface, dynamics condition is

$$p_W=p_{A\infty}+\rho_1 gh-\rho_2 gw+\rho_2 U\partial\phi/\partial x \tag{6}$$

At infinity, w is also determined by following equations

$$w=0,\ \nabla w=0 \qquad |x|\to\infty \tag{7}$$

Combining Eq. (1) and Eqs. (5)–(7), we have

$$\frac{D}{\rho_2 g}\nabla^4\left(\int_x^{+\infty}\frac{\partial\phi}{\partial z}\mathrm{d}x\right)+\frac{\tau UD}{\rho_2 g}\nabla^4\left(\frac{\partial\phi}{\partial z}\right)+\int_x^{+\infty}\frac{\partial\phi}{\partial z}\mathrm{d}x-\frac{U^2}{g}\frac{\partial\phi}{\partial x}-\frac{\rho_1 hU}{\rho_2 g}\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial z}\right)=-\frac{U}{\rho_2 g}p_{Ae} \tag{8}$$

Thus, we can use Eq. (8) and Eqs. (2)–(4) to solve the ice-water vibration problem induced by ACV.

When h=0, Eq. (8) reduces to Eq. (4), so Eqs. (2)–(4) can be used to solve the wave-making issue of pure water.

When h≠0and D=0, Eq. (8) reduces to

$$\int_x^{+\infty}\frac{\partial\phi}{\partial z}\mathrm{d}x-\frac{U^2}{g}\frac{\partial\phi}{\partial x}-\frac{\rho_1 hU}{\rho_2 g}\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial z}\right)=-\frac{U}{\rho_2 g}p_{Ae} \tag{9}$$

In this case, the Eq. (9) is equivalent to the Eq. (1.2) from Kozin and Milovanova (1996). Thus, the Eq. (9) and Eqs. (2)–(3) can be used to solve broken ice problem.

The distribution of surface pressure can be written in the form of multiplication of hyperbolic tangent function (Doctors and Sharma, 1972), i.e.

$$p_A=\frac{p_0}{4}\left\{\tanh\left[\alpha\left(x-x_0-\frac{L}{2}\right)\right]-\tanh\left[\alpha\left(x-x_0+\frac{L}{2}\right)\right]\right\}\cdot\left\{\tanh\left[\beta\left(y-y_0-\frac{B}{2}\right)\right]-\tanh\left[\beta\left(y-y_0+\frac{B}{2}\right)\right]\right\} \tag{10}$$

where p_0 is the nominal pressure, L is the length of ACV, B is the width of ACV, (x_0, y_0) is the center position of ACV, α and β are the parameters which control the rate of pressure fall-off at the edges of ACV.

After obtaining the disturbance potential ϕ, the vertical deflection of free surface or ice sheet can be calculated by following formula

$$w(x,y)=\frac{1}{U}\int_x^{+\infty}\frac{\partial\phi}{\partial z}\mathrm{d}x \tag{11}$$

The formula for the wave resistance can be written as follows

$$R_W=\iint_S p_{Ae}\frac{\partial w}{\partial x}\mathrm{d}x\mathrm{d}y=-\frac{1}{U}\iint_S p_{Ae}\left(\frac{\partial\phi}{\partial z}\right)_{z=0}\mathrm{d}x\mathrm{d}y \tag{12}$$

where S is the area of pressure distribution.

Normal stress σ_{xx}, σ_{yy} and shear stress σ_{xy} within the ice sheet can be expressed as follows

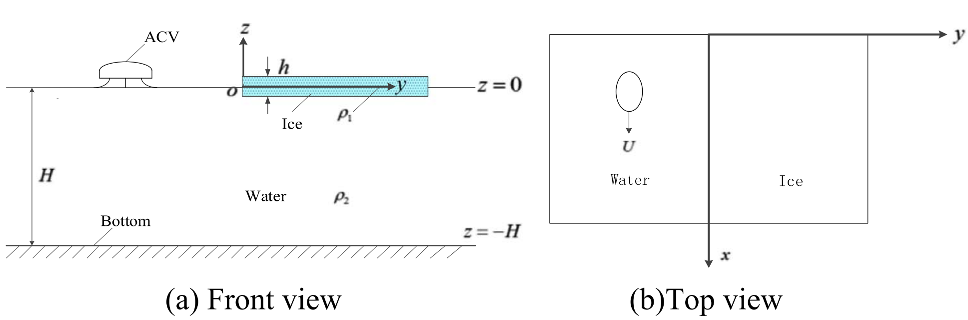

Fig. 1. Sketch of mathematical model.